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DEVELOPMENT AND TIME DOMAIN VALIDATION  
OF A LOW-ORDER, CONSTANT SPEED, PITCH-HEAVE  
MODEL FOR THE XR-3 SURFACE EFFECT SHIP

Leslie William Barnes

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## Monterey, California



# THESIS

Development and Time Domain Validation  
of a Low-Order, Constant Speed, Pitch-Heave  
Model for the XR-3 Surface Effect Ship

by

Leslie William Barnes

March 1979

Advisor:

A. Gerba

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DEVELOPMENT AND TIME DOMAIN VALIDATION OF A LOW-ORDER,  
CONSTANT SPEED, PITCH-HEAVE MODEL FOR THE XR-3 SURFACE  
EFFECT SHIP

by

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Submitted in partial fulfillment of the  
requirements for the degree of

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## ABSTRACT

An improved and simplified model for the Heave-Pitch dynamics of the XR-3 CAB Surface Effect Ship is developed for constant speed operation. The nonlinear equations of motion are linearized about the steady-state operating point. Time-domain validation is accomplished by comparison with the 6 DOF model. A signal flow graph of the craft dynamics is developed and Mason's Gain Rule used to determine the characteristic S-Domain polynomials describing the craft's vertical plane dynamic behavior, with Bode Plot analysis included. Conclusions are drawn and recommendations for further study are made.



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## I. INTRODUCTION

In 1973 Leo and Boncal [1] converted the Oceanics Inc. 6 DOF Loads and Motions digital computer simulation program for the 100-B Surface Effect Ship to reflect the physical characteristics of the XR-3 craft. Since that time there has been a continuous effort at the NPS to update and improve on this simulation [2,3,4,5].

One of the major drawbacks of that program which precludes regular use of it for sea state operations simulation is the excessive computational and turn-around times required [6]. As it is presently desired to use the 6 DOF simulation for design and evaluation studies of the proposed 3K Ton SES control System in varying sea state conditions, it is important to significantly reduce the simulation's computational time.

One area where the possibility exists to achieve a major reduction in computational time is the manner in which sidewall forces and moments are calculated. The present form of the 6 DOF program uses a large number of discrete sections (28 per sidewall) with an off-line table look-up scheme to determine the forces generated in each discrete section. The simple model derived by McIntyre [7] uses two sections per sidewall with very simplified geometry that permits on-line computation of the forces and moments. The result of eventual incorporation of this sidewall geometry and forces calculation into the 6 DOF program should be a significant reduction of computation and turn-around time.

Another area of great interest in SES Simulation is





gaining insight into the dynamic behavior of the craft's vertical motion of pitch and heave due to pressure induced lift forces generated in the plenum and seals. As stated in ref [8] as recently as August, 1978,

'The overall precision of the analysis is satisfactory for predicting stability and maneuverability. However, several areas present very complex or difficult physical phenomena to analyze and further effort should be invested to improve understanding of these.'

The first area for continuing investigation recommended was;

'A better determination of pitch-heave dynamic characteristics.'

An improved knowledge of the dynamic characteristics brought about by the pressure induced forces and moments could lead to a reduction of the computation time required for craft simulation. These pressure effects introduce a stiffening in the integration of sea state induced oscillations and a better understanding of processes could lead to the removal of this stiffness and thus reduced CPU time.

In this work both of these areas are considered. First of all the various craft physical characteristics are modeled, resulting in changes and improvements to the initial efforts by McIntyre [7]. These equations are linearized about a steady-state operating point using the Taylor Series expansion. The linearized equations are cast into a State Space format and simulated on the digital computer. Comparison is made between the time-domain results of the linear model and the 6 DOF model. The State Space representation is used to generate a signal flow graph. Mason's Gain Rule is



applied and the S-domain roots of the craft's characteristic equations are located and it is shown that some simplification and reduction of system order is possible. The resultant S-domain polynomials are used to generate Bode Plots to graphically display the craft's simplified frequency response.

Conclusions are drawn and recommendations are made concerning the validity of the model and areas for future study.

Several procedural notes are in order. Because all equations are to be solved digitally using the FORTRAN computer language and because digital computer text processing facilities were used to generate this document, all formulas will be presented in FORTRAN format. In the thesis body reference to the "nonlinear model", "six degree-of -freedom model" or "6 DOF" model all mean the Loads and Motions program as adapted to the XR-3 test craft at the Naval Postgraduate School [3]. Reference to the linear model refers to the linearized model of the same craft as developed herein.



## II. NONLINEAR XR-3 MODEL HEAVE AND PITCH EQUATIONS

### A. BACKGROUND

The original development work on the simplified two-degree-of-freedom model was done by McIntyre [7]. Several weaknesses in the original model were noted by McIntyre in that work.

The major portion of the work presented here was involved in close examination of all aspects of the original model, comparison of the modeling techniques used therein with other methods and remodeling of several important craft physical characteristics.

One major area of investigation was the lack of pitch damping in the original model, which resulted in the addition of an added-mass effect to the pitch moment equations.

Additional changes involve a revision of the force equations for the rear seal, major changes in the way the plenum chamber is modeled (also added to the loads and motion 6-dof model [2]). and a more rigorous development of the planing force equation of [7].

There are several areas of theory concerning CAB SES modeling where empirical data is lacking due to difficulties with physical measurements. One such area inviting closer inspection was dynamic variation of the



effective center of pressure for the plenum gauge pressure lift effect. A possible approach is suggested herein and included in the model.

## B. ASSUMPTIONS

The following assumptions are made for this constant speed, pitch-heave model:

1. Planing forces are concentrated at the centroid of the keel longitudinal cross section.

2. Total seal force is lumped into a pressure differential acting on the seal face area in contact with the water passing beneath the seal.

3. Aerodynamic forces on the forward seal are disregarded.

4. Dynamic variations in moment lever arms are disregarded for all moments except seals and the moment due to plenum gauge pressure.

5. Dynamic variations in effective plenum roof area will be included.

6. Any lift effects due to aerodynamic forces are lumped in with the planing forces (for reasons to be developed in Chapt IV.) The moments are included in plenum gauge pressure moments .

7. Constant seal leakage area is assumed.

8. Since this model is for constant surge velocity,





drag and thrust forces are considered to be in equilibrium and in balance for moments about the Y axis.

### C. COORDINATE SYSTEM

(see fig. 1 )

1. The origin of the cartesian coordinate system is located at the calm-water line, longitudinally and transversely coincident with the craft center of gravity.

2. The X axis is the longitudinal coordinate defined as positive as X increases towards the bow of the craft.

2. The Y axis is the transverse coordinate increasing positively towards the starboard side.

4. The Z axis is the vertical axis, increasing positively in a downward direction.

5. ZS is the vertical distance from the center of gravity to the keel.

6. Z is the vertical distance from the calm water line to the vertical center of gravity.

7. THETA is the pitch angle defined as positive as the bow pitches up.

8. Positive pitch moments are defined as moments which tend to cause a positive increase in the angle THETA.

### D. SIDEWALL MODELING



As it was desired to keep all physical dimensions consistent with the six degree of freedom model, scale model drawings were developed with the data from the sidewall subroutine of that model. All sidewall dimensions used in this model were taken from those drawings.

The sidewalls are modeled in two sections, one forward of the longitudinal C.G. and the other aft. Each sidewall is considered to be of constant width at the water line and the keel, with width increasing in the vertical direction according to an average deadrise angle.

The average forward sidewall area is found to be:

(see fig. 2 and 3b )

$$\text{AVERAGE WIDTH} = \text{LDBAR1} / (2 * \text{TAN}(\text{DR1})) + \text{WS10}$$

(II-1)

where LDBAR1 is draft at forward centroid, DR1 = average deadrise angle forward and WS10 is the average keel width of the forward section. Therefore submerged volume of a forward sidewall is given by:

$$\text{VOL} = \text{L1} * (\text{LDBAR1} / (2 * \text{TAN}(\text{DR1})) + \text{WS10}) * \text{LDBAR1}$$

(II-2)

where L1 is the length of a forward buoyant section.

Following the same procedures the after sidewall width



is:

(see fig. 2 and 3c )

$$\text{AVERAGE WIDTH} = \text{LDBAR2} / (2 * \text{TAN}(\text{DR2})) + \text{WS20}$$

(II-3)

where DR2 = average deadrise angle aft

LDBAR2 = draft at after buoyant centroid

WS20 = average keel width aft

Submerged volume of an after sidewall is then determined to be;

$$\text{VOL} = \text{L2} * (\text{WS20} + \text{LDBAR2} / (2 * \text{TAN}(\text{DR2}))) * \text{LDBAR2}$$

(II-4)

where L2 is the length of the after sidewall.

## E. SEAL MODELING

A major change in McIntyre's model was made by remodeling the stern seal. In the original model a large negative moment was required to place the pitch equations in steady state equilibrium. Investigation of seal forces as presented in [9] revealed a large discrepancy between the after seal forces generated in McIntyre's model and calculations based on the information in [9]. This discrepancy was further supported by actual seal load data presented by Layton in [10]. All seal dimensions used in this model were taken from data presented in [10]. Seal hinge locations were taken from the data used in [5].



Seal lift force forward is generated by plenum gauge pressure acting downward on the seal face area in contact with the water passing underneath the seal. This downward force results in a lift force being generated as the downward movement of the seal displaces the water beneath it.

The seal is considered to have a straight forward face which runs from the hinge point at the plenum roof to the bottom of the keel at an angle determined to be best-fit from the data in ref. [1]. The wetted surface of the seal is considered to lay parallel to and on the water at the point where the seal face intercepts the calm water line. as a result forward seal lift equations are:

$$ASEAL1 = WIDTH * XSEAL1$$

(II-5)

$$XSEAL1 = (LD - L3 * \tan(\theta)) / \sin(31)$$

(II-6)

LD = draft at the C.G

L3 = distance from C.G to point of contact for forward seal face.

$LD - L3 * \tan(\theta)$  = draft at forward seal face contact point

31 degrees is the interior angle of the seal.

The after seal wetted area is determined in a similar manner to be :

$$ASEAL2 = WIDTH * XSEAL2$$





(II-7)

$$XSEAL2 = (LD + L4 * \tan(\theta)) / \sin(32)$$

(II-8)

In this model forward seal pressure is considered equal to plenum pressure. As a result forward seal lift is :

$$HSEALF = PBBAR * ASEAL1$$

(II-9)

Since plenum guage pressure tends to force the after seal upward, resulting in venting of the plenum air mass, the rear seal is operated at a pressure slightly above that of the plenum to reduce the venting tendency of the after seal. As a result the after seal requires a modification in modeling. In McIntyre's work the pressure differential ( $PSEAL - PBBAR$ ) only was considered in the lift force. However, as previously stated, there were large discrepancies in computed loads as compared to measured data from [10]. Therefore in this model, the seal working pressure is considered to be  $PBBAR$  plus differential pressure. As a result after seal lift is given by:

$$HSEALA = (PBBAR + PDIFF) * ASEAL2$$

(II-10)



## F. PLENUM MODELING

In the work presented by McIntyre the plenum was considered a box with the only dynamic dimensional variation being in the Z direction. However (as can be seen in fig. 1), although the forward edge can be assumed to be reasonably constant in the Y-Z plane, the forward face of the rear seal slopes rather steeply. Therefore the plenum length (in the X direction) varies rapidly as draft varies. As a result the effective plenum roof area consists of that area from the after edge of the forward seal to the forward hinge point of the rear seal, plus the vertical projection of the unwetted forward face of the rear seal. As an additional consequence it can be seen that the effective plenum center of pressure varies fore and aft as draft varies. In fact as draft increases the C.P. moves forward.

The following variable names will be used to describe the plenum chamber (see fig. 4.)

LPR = length of plenum roof from the rear edge of the forward seal to the hinge point of the after seal.

LPWL = length of plenum at the keel from the after edge of the forward seals to the transom.

LP = length of plenum at the waterline.

BUBHGT = vertical distance from the plenum roof to the bottom of the keel at the center of gravity.

AB = plenum effective roof area.

Using the previous definitions, the plenum roof length and area is:



$$LP = LPWL - (LPWL - LPR) * (LD / BUBHGT)$$

(II-11a)

and effective plenum roof area is

$$AB = WIDTH * LP$$

(II-11b)

with the result that lift due to plenum gauge pressure is:

$$HPRES = -AB * PBBAR$$

(II-11c)

As a comment on the complexity of the lift forces of this model note that for constant plenum pressure increased draft results in decreased plenum lift.

## G. CRAFT DYNAMICS

### 1. Plenum Air Mass and Pressure Equations

For the purposes of this model the rear seal leakage area is considered constant since to assume otherwise would lead to modeling a very difficult and poorly understood dynamic process. In addition the plenum pressure VS air mass process is assumed to be adiabatic in nature. Fan characteristics are those used in the constant air leakage heave only model of Gerba and Thaler in [14]. Air flow into the plenum is taken as positive.



Therefore the airflow into the chamber is given by:

$$Q_{in} = N \cdot (Q_{I0} - K_q \cdot P_{BBAR})$$

(II-12)

Where N is the number of fans supplying air to the plenum,  $Q_{I0}$  is the equilibrium air flow rate for the fan with zero gauge pressure and  $K_q$  is the slope of the fans' airflow VS gauge pressure curve .

$Q_{out}$  is airflow out of the plenum and is determined by the seal leakage equation:

$$Q_{out} = C_n \cdot A_l \cdot (2 \cdot P_{BBAR} / \rho_{HOA})^{1/2}$$

(II-13)

Where  $C_n$  = seal leakage coefficient,  $A_l$  = seal leakage area, and  $\rho_{HOA}$  = mass density of air. Therefore mass flow rate is:

$$\dot{M}_{BDOT} = \rho_{HOA} \cdot (Q_{in} - Q_{out})$$

(II-14)

The plenum air dynamics are considered in equilibrium when  $Q_{in} = Q_{out}$

Absolute Plenum Pressure is determined from the adiabatic gas law to be:

$$P_b = P_a \cdot (\dot{M}_B / (\rho_{HOA} \cdot P_{BBAR}))^{1/\gamma}$$





(II-15)

and,

$$PBBAR = P_b - P_a$$

(II-15a)

Where  $P_a$  = ambient atmospheric pressure (LBF/FT\*\*2  $\gamma$ )  
= adiabatic process coefficient,  $P_b$  = plenum absolute pressure, and PBBAR = plenum gauge pressure.

## 2. Buoyant Forces

Buoyant force is calculated by the submerged volume multiplied by the mass density of the fluid times the gravitational constant.

Therefore forward buoyant force is:

$$HBF = -2 * RHO * G * VOL$$

(II-16)

Where volume is given by eqn. II-2,  $RHO$  is the mass density of water (slugs/ft\*\*3) and  $G$  is the gravitational constant. The factor of 2 is introduced to account for 2 sidewalls. The total expression for HBF is:

$$HBF = -2 * RHO * G * L1 * LDBAR1 * (LDBAR1 / (2 * TAN(DR1))) + WS01$$

(II-17)

Following the same pattern,



$$HBA = -2 * RHO * G * L2 * LDBAR2 * (LDBAR2 / (2 * TAN(DR2))) = WS10)$$

(II-18)

### 3. Seal Forces

Forward seal force is determined from eqn. II-9 to be:

$$HSF = -PBBAR * WIDTH * (LD - L3 * TAN(THETA)) / SIN(31)$$

(II-19)

where HSF is the heave force due to forward seal.

In a similar manner HSA is:

$$HSA = (PBBAR + PDIFF) * WIDTH * (LD + L4 * TAN(THETA)) / SIN(32)$$

(II-20)

### 4. Planing Forces

McIntyre introduced the planing force phenomena in his model to account for the proven fact that the CAB SES tends to decrease it's draft and pitch down as speed increases. His development of the planing equation was not well explained and the basis for selection of the planing coefficient was not clearly stated. Reidel [12] did a special investigation of the planing force phenomena. His results were based on flat plate planing effects to approximate the flat bottom of the craft sidewalls.



Since the water is in contact with the vertical portions of the sidewalls it was felt a more accurate representation was needed. Barnaby [13] states that water contact on the vertical sides of a planing hull rapidly reduces planing lift advantage since one of the advantages of a planing hull is reduced drag due to wetted surface. planing lift results in a speed sensitive tradeoff between static lift (buoyant effects) and dynamic lift (planing effect) that is directly proportional to the angle of attack (for small planing angles) and the square of the velocity. Therefore the planing force is of the form:

$$HPLAN = K * V^{**2} * THETA$$

From Reidel's work,

$$HPLAN = 0.5 * AREA * V^{**2} * PI * SIN(THETA)$$

(II-21)

for small angle approximations this simplifies to:

$$HPLAN = RHO * AREA * V^{**2} * PI * THETA$$

Which is the planing force for a flat plate. Barnaby gives no simple formula for the losses due to immersion so we are left with an arbitrary loss coefficient, PLCOEF such that:

$$HPLAN = (PLCOEF) * 2 * RHO * AREA * V^{**2} * PI * THETA$$

(II-22)

The added factor of 2 accounts for 2 sidewalls.

The planing area is taken from the scale drawings



developed from sidewall data, and the force is assumed to act at the centroid of the flat keel surface, which is consistent with Reidel's work. The loss coefficient will be developed in the section on achieving equilibrium.

## 5. Plenum Gauge Pressure Lift

Combining equations II-11a , II-11b , and II-11c results in:

$$HPRES = -PBBAR*WIDTH*(LPWL - (LPWL - LPR)*(LD/BUBHGT))$$

(II-23)

## 6. Total Lift Forces

$$Lift\ Total = HPRES + HBF + HBA + HSF + HSA + HPLAN$$

which must exactly equal craft weight (W) for the hull to be in vertical equilibrium, or;

$$W + HPRES + HBF + HBA + HSF + HSA + HPLAN = 0$$

(II-24)

defines equilibrium condition in heave motion.

## H. CRAFT PITCH MOMENT EQUATIONS

### 1. Buoyant Moments





Forward buoyant moment is the forward buoyant lift times the effective lever arm, or;

$$PBF = -HBF * (\text{Lever arm forward})$$

$$PBF = -HBF * L5$$

(II-25)

Note that the negative sign results from the definition of positive moments and the fact that upward forces are negative by definition.

$$PBA = HBA * \text{LEVER ARM}$$

$$PBA = HBA * L6$$

(II-26)

using the same reasoning, after buoyant moment is:

$$PB \text{ total} = PBF + PBA$$

(II-27)

## 2. Seal Moments

Since the seals are relatively distant from the C.G. dynamic variations in the lever arms are considered in seal moment equations and are developed here.



The seal force is modeled to act at the longitudinal center of the seal wetted length. Therefore the effective lever arm for the seal moment is:

$$PLSF = L3 - XSEAL1/2$$

where PLSF = moment lever arm for seal forward. As a result:

$$PSF = -HSF*PLSF$$

$$PSF = -HSF*(L3-(XSEAL1/2))$$

(II-28)

In a similar manner

$$PLSA = L4 + XSEAL2/2$$

$$PSA = HSA*PLSA$$

$$PSA = HSA*(L4+XSEAL2/2)$$

(II-29)

$$PS\ TOTAL = PSF+PSA$$

Note that the dynamic pitch sensitivity to moment arm length is included in the XSEAL terms as defined in equations II-6 and II-8.



### 3. Planing Moment

For the planing force developed in equation II-22 the planing force is concentrated at the centroid of the flat keel which results in:

$$PPLAN = L7 * HPLAN$$

(II-30)

### 4. Plenum Pressure Moment

First the dynamic variation in effective center of pressure will be developed. (see fig.(4))

plenum length is taken from equation II-11a to be

$$LP = LPWL - (LPWL - LPR) * (LD / BUBHGT)$$

Defining LDIFF to be

$$LDIFF = LPWL - LPR$$

(II-31)

and,

$$XCP = XCP0 + (LDIFF * LD / BUBHGT) / 2$$

(II-32)

where



$$XCP0 = LPWL/2 - XCG$$

(II-33)

and XCG is the distance from the transom to the C.G.  
(see fig. 4 .) Finally,

$$PPRES = -HPRES * XCP$$

(II-34)

## 5. Pitch Damping Moment

Any frictional damping terms dependent upon the square of pitch rate (i.e. tangential velocity about the Y axis) disappear when the Taylor Series expansion is carried about the zero rate equilibrium point. Since for small signal approximation the model should reflect actual craft characteristics, it was hoped that the changes in seal and plenum modeling would reveal the missing damping characteristics. As it turned out this was not the case. Examination of the hydrodynamic equations in [9] revealed an added mass effect dependant upon a cross velocity term between linear X direction velocity U, pitch rate Q, and added-mass at the stern A33S due to slender body theory applied to the sidewalls. This moment is:

$$FP = -A33S * XLSS ** 2 * U * THETADOT$$

(II-35)

Where XLSS is the distance from the C.G. to the stern, and THETADOT is pitch rate.





$$A33S = YSS^{**2} * RHO * PI / 8$$

(II-36)

where YSS is the cross section width of the side wall at the stern waterline. Therefore,

$$FPDAMP = -2 * XLSS^{**2} * A33S * U * THETADOT$$

$$= -2 * PI * XLSS^{**2} * YLSS^{**2} * RHO * U * THETDOT / 8$$

(II-37)

The factor of 2 is introduced to account for two sidewalls.

## 6. Total Moments

Since the moments are all referenced to the craft center of gravity, the craft weight contributes zero moment in that frame of reference. Therefore total moments are:

$$PSF + PSA + PBF + PBA + PRES + PPLAN + PDAMP = 0$$

(II-38)

## I. HEAVE ACCELERATION EQUATIONS.

From Newton's law, acceleration and force are related by:



$$\text{FORCE} = \text{MASS} * \text{ACCELERATION}$$

For the linearized model we are interested in the acceleration on the craft and as a result:

$\text{ACCELERATION} = \text{FORCE} / \text{MASS}$  and therefore the heave equation of motion becomes:

$$\text{ZDDOT} = (\text{HBF} + \text{HBA} + \text{HSA} + \text{HPLAN} + \text{HPRES}) / \text{MASS}$$

(II-39)

where ZDDOT is the second derivative of Z with respect to time.

#### J. PITCH ACCELERATION EQUATIONS

Angular acceleration is:

$$\text{THETADDOT} = \text{PTOTAL} / \text{IYY}$$

Where IYY is the craft inertial moment about the Y axis. As a result total angular acceleration is;

$$\text{THETADDOT} = (\text{PBF} + \text{PBA} + \text{PSF} + \text{PSA} + \text{PPLAN} + \text{PPRES} + \text{PDAMP}) / \text{IYY}$$

(II-40)

This completes the derivation of all nonlinear equations needed to describe the two-degree-of-freedom CAB SES. In the following chapter these nonlinear equations will be linearized using the Taylor Series expansion about a steady state operating point which will result in the equations needed to generate a



dynamic digital-computer simulation of craft motion in  
the time-domain. .



### III. LINEARIZATION OF THE HEAVE-PITCH EQUATIONS.

#### A. TAYLOR SERIES EXPANSION

For small perturbations about a steady state operating point the nonlinear function  $Z$  can be approximated by:

$$Z = Z(X,Y)$$

$$Z(0)+dZ = Z(X(0),Y(0)) + (\partial Z/\partial X) *dX + (\partial Z/\partial Y) *dY$$

where  $(\partial Z/\partial X)$  and  $(\partial Z/\partial Y)$  are the partial derivatives of the function  $Z$  evaluated at the steady state operating point  $X(0),Y(0)$ . Cancelling the steady state values from both sides of the equation yields the differential model equations:

$$dZ = (\partial Z/\partial X) *dX + (\partial Z/\partial Y) *dY$$

and

$$Z = Z(0)+dZ$$

where  $dZ, dX$ , and  $dY$  are the differential variables for  $Z$ ,  $X$ ,  $Y$  respectively,  $(\partial Z/\partial X)$  and  $(\partial Z/\partial Y)$  are the partial derivatives of  $Z$  evaluated at the steady-state operating point.





## B. LINEARIZATION OF THE MODEL EQUATIONS

There are basically four equations to linearize for the two-degree-of-freedom model:

$$\ddot{Z} = F(Z, \theta, \dot{M})$$

$$\ddot{\theta} = G(Z, \theta, \dot{\theta}, \dot{M})$$

$$\dot{M} = H(Z, \dot{M})$$

$$\bar{P} = I(Z, \dot{M})$$

therefore the equations for the differential state variables are:

$$\dot{Z} \ddot{Z} = \left( \frac{dF}{dZ} \right) \dot{Z} + \left( \frac{dF}{d\theta} \right) \dot{\theta} + \left( \frac{dF}{d\dot{M}} \right) \dot{M} \quad (\text{III-1})$$

$$\dot{\theta} \ddot{\theta} = \left( \frac{dG}{dZ} \right) \dot{Z} + \left( \frac{dG}{d\theta} \right) \dot{\theta} + \left( \frac{dG}{d\dot{M}} \right) \dot{M} \quad (\text{III-2})$$

$$\dot{M} \ddot{M} = \left( \frac{dH}{dZ} \right) \dot{Z} + \left( \frac{dH}{d\dot{M}} \right) \dot{M} \quad (\text{III-3})$$



$$dPBBAR = (dI/dZ) * dZ + (dI/dTHETA) * dTHETA + (dI/dMB) * dMB$$

(III-4)

Note that the left hand side variables are differentials, not time derivatives.

## C. LINEARIZED EQUATIONS

### 1. Plenum Air Mass Flow Derivatives

$$Q_{in} = N * (Q_{i0} - K_q * PBBAR)$$

(II-12)

$$Q_{out} = C_n * A_1 * (2 * PBBAR / \rho_{HOA})^{**1/2}$$

(II-13)

$$MBDOT = \rho_{HOA} * (Q_{in} - Q_{out})$$

(II-14)

as a result the differential equation for MBDOT becomes:

$$dMBDOT = \rho_{HOA} * (dQ_{in} - dQ_{out})$$

(III-5)

$$dQ_{in} = (dQ_{in}/dZ) * dZ + (dQ_{in}/dPBBAR) * dPBBAR + (dQ_{in}/dMB) * dMB$$



(III-6)

$$dQ_{in}/dP_{BBAR} = -N \cdot K_1$$

The partial derivatives of  $Q_{in}$  with respect to all other variables are zero.

$$dQ_{out} = (dQ_{out}/dz) \cdot dz + dQ_{out}/dP_{BBAR} \cdot dP_{BBAR} + (dQ_{out}/dMB) \cdot dMB$$

$$dQ_{out}/dP_{BBAR} = C_n \cdot A_1 \cdot ((\rho_{HOA}/2 \cdot P_{BBAR})^{**1/2}) / \rho_{HOA}$$

(III-7)

The partial derivatives of  $Q_{out}$  with respect to all other variables are zero. Inserting equations III-6, and III-7 into II-14 yields the equation for the differential variable  $dMBDOT$ :

$$dMBDOT = (-\rho_{HOA} \cdot N \cdot K_1 - C_n \cdot A_1 \cdot ((\rho_{HOA}/2 \cdot P_{BBAR})^{**1/2})) \cdot dP_{BBAR}$$

(II-15)

## 2. Plenum Gauge Pressure Derivatives.

$$P_b = P_a \cdot (M_b / \rho_{HOA} \cdot V_b)^{**GAMMA}$$

(III-8)

and

$$P_{BBAR} = P_b - P_a$$



Since  $P_a$  is considered constant within the time frame of this model,

$$dP_{BBAR} = dP_b$$

(III-9)

As a result,

$$dP_{BBAR} = \text{GAMMA} * P_a * (MB(0) / \text{RHOA} * V_b(0)) ** (\text{GAMMA} - 1) * d(MB / \text{RHOA} * V_b)$$

where

$$d(MB / \text{RHOA} * V_b) = (1 / \text{RHOA} * V_b(0)) * dMB + (MB / \text{RHOA}) * d(1 / V_b)$$

$$d(MB / \text{RHOA} * V_b) = (1 / \text{RHOA} * V_b(0)) * dMB - (MB / \text{RHOA} * V_b ** 2) * dV_b$$

$$dV_b = d(VN - AB * LD)$$

$$dV_b = -AB(0) * dLD / dZ - LD(0) * dAB / dZ$$

remembering that:

$$LD = Z + ZS$$

then

$$dLD / dZ = 1$$

we now need the partial of effective plenum roof area with respect to  $Z$ . Defining





$$AB = ABWL - (ABWL - ABR) * LD / BUBHGT$$

(III-10)

$$ABWL = WIDTH * LPWL$$

$$ABR = WIDTH * LPR$$

further defining

$$ABDIFF = ABWL - ABR$$

which is a constant, the effective plenum roof area is:

$$AB = ABWL - ABDIFF * LD / BUBHGT$$

using the previous definitions the partial of effective roof area becomes:

$$dAB/dZ = -ABDIFF / BUBHGT$$

(III-11)

and the differential volume is:

$$dVb = - (AB(0) - LD(0) * 2 * (ABDIFF / BUBHGT)) * dZ$$

(III-12)

with the result that

$$d(Mb / RHOA * Vb) = (1 / RHOA * Vb(0)) * dZ +$$

$$(Mb(0) / RHOA * Vb(0) ** 2) * (AB(0) - 2 * LD(0) * (ABDIFF / BUBHGT)) * dZ$$



finally,

$$\begin{aligned} dPBBAR = & GAMMA * Pb(0) * ((1 / Mb(0)) * dMb + \\ & (1 / Vb(0) * (AB(0) - 2 * LD(0) * (ABDIFF / BUBHGT))) * dz \end{aligned}$$

(III-13)

### 3. Heave Force Derivatives

The total heave force equation is:

$$H(\text{total}) = HBF + HBA + HSF + HSA + HPLAN + HPRES$$

The total partial derivatives will be done term by term.

For the forward buoyant force the following simplifications are made:

$$HBF = -2 * RHO * G * L1 * LDBAR1 * (LDBAR1 / (2 * TAN(DR1)) + WS10)$$

let  $K1 = 2 * RHO * G * L1$  then,

$$HBF = -K1 * LDBAR1 * (LDBAR1 / (2 * TAN(DR1)) + WS10)$$

where  $LDBAR1 = (LD - L5 * TAN(THETA))$ . Combining the previous two equations,

$$\begin{aligned} HBF = & -K1 * (LD - L5 * TAN(THETA)) * \\ & ((LD - L5 * TAN(THETA)) / (2 * TAN(DR1)) + WS10) \end{aligned}$$



assuming small pitch angles,  $\text{TAN}(\text{THETA})$  can be approximated by  $\text{THETA}$  which results in the following simplification:

$$\text{HBF} = -K1 * (\text{LD} - \text{L5} * \text{THETA}) * ((\text{LD} - \text{L5} * \text{THETA}) / (2 * \text{TAN}(\text{DR1})) + \text{WS10})$$

Taking the partials of HBF with respect to  $Z$  and  $\text{THETA}$  yields:

$$\frac{d(\text{HBF})}{dZ} = -K1 * ((\text{LD}(0) - \text{L5} * \text{THETA}(0)) / \text{TAN}(\text{DR1}) + \text{WS10}) \quad (\text{III-14})$$

and

$$\begin{aligned} \frac{d(\text{HBF})}{d\text{THETA}} = & K1 * \text{L5} * ((\text{LD}(0) - \text{L5} * \text{THETA}(0)) / (2 * \text{TAN}(\text{DR1})) + \text{WS10}) \\ & + K1 * (-\text{L5} / (2 * \text{TAN}(\text{DR1})) * (\text{LD}(0) - \text{L5} * \text{THETA}(0))) \end{aligned}$$

Which simplifies to :

$$\begin{aligned} \frac{d(\text{HBF})}{d\text{THETA}} = & K1 * \text{L5} * ((\text{LD}(0) - \text{L5} * \text{THETA}(0)) / (\text{TAN}(\text{DR1})) + \text{WS10}) \end{aligned} \quad (\text{III-15})$$

Using the same approach, the partials for buoyancy force aft are:

$$\frac{d(\text{HBA})}{dZ} = -K2 * ((\text{LD}(0) + \text{L6} * \text{THETA}(0)) / (\text{TAN}(\text{DR2})) + \text{WS20}) \quad (\text{III-16})$$



$$\begin{aligned} d(HBF)/dTHETA = \\ -L6*K2*((LD(0) + L6*THETA(0)/(TAN(DR2)) + WS10) \end{aligned}$$

(III-17)

For the forward seal the total differentials are:

$$\begin{aligned} DHSF = (d(HSF)/dZ) * dZ + (d(HSF)/dTHETA) * dTHET \\ + (d(HSF)/dPBBAR) * dPBBAR \end{aligned}$$

the following simplification is made;

$$HSF = -PBBAR*WIDTH*(LD-L3*TAN(THETA)/SIN(31))$$

(II-19)

let  $WIDTH/SIN(31) = K3$  and  $TAN(THETA) = THETA$ , then,

$$HSF = -K3*PBBAR*(LD-L3*THETA)$$

with the resultant partials being:

$$d(HSF)/dZ = -K3*PBBAR(0)$$

(III-18)

$$d(HSF)/dTHETA = K3*L3*PBBAR(0)$$

(III-19)

$$d(HSF)/dPBBAR = -K3*(LD(0) - L3*THETA(0))$$

(III-20)





The partials of after seal force are found in the same manner to be:

$$\begin{aligned} dHSA = & (d(HSA)/dZ) * dZ + (d(HSA)/dTHETA) * dTHETA \\ & + (d(HSA)/dPBBAR) * dPBBAR \end{aligned}$$

$$HSA = - (PBBAR + PDIFF) * WIDTH * (LD + L4 * TAN(THETA) / SIN(32)) \quad (II-20)$$

let  $K4 = WIDTH / SIN(32)$  and assume small THETA so that;

$$HSA = -K4 * (PBBAR + PDIFF) * (LD + L4 * THETA)$$

the partials then become:

$$d(HSA)/dZ = -K4 * (PBBAR(0) + PDIFF) \quad (III-21)$$

$$d(HSA)/dTHETA = -K4 * (PBBAR(0) + PDIFF) * L4 \quad (III-22)$$

$$d(HSA)/dPBBAR = -K4 * (LD(0) + L4 * THETA(0)) \quad (III-23)$$

Next the planing force partials are derived:

$$HPLAN = -2 * PLCOEF * RHO * AREA * V ** 2 * PI * THETA$$

let  $KPLAN = (2 * PLCOEFF * RHO * AREA * PI)$  then,



$$HPLAN = -KPLAN * V^{**2} * THETA$$

and,

$$d(HPLAN) = (d(HPLAN)/dTHETA) * dTHETA$$

for this constant speed model, therefore,

$$d(HPLAN)/dTHETA = -KPLAN * V(0)^{**2}$$

(III-24)

Plenum Gauge Pressure lift was defined to be:

$$HPRES = -AB * PBBAR$$

(II-11c)

as a result:

$$d(HPRES)/dPBBAR = -AB(0)$$

(III-25)

#### 4. Pitch Moment Partials

The total moments acting on the craft are:

$$PTOTAL = PBF + PBA + PSF + PSA + PPLAN + PPRES + PDAMP$$

(II-38)

The partials of II-38 will be found term by term, beginning with the forward buoyant moment:



$$PBF = -HBF * L5$$

(II-23)

therefore:

$$d(PBF) = -L5 * d(HBF)$$

so that:

$$d(PBF) = -L5 * (d(HBF) / dZ) * dZ - L5 * ((d(HBF) / dTHETA) * dTHETA$$

(III-26)

and the partial of the after buoyant moment is:

$$PBA = L6 * HBA$$

(II-26)

$$dPBA = L6 * dHBA$$

therefore,

$$d(PBA) = L6 * (d(HBA) / dZ) * dZ + L6 * (d(HBA) / dTHETA) * dTHETA$$

(III-27)

Seal forces partials must include the dynamic variation in lever arm length, which for the forward seal is:

$$PSF = -HSF * PLSF$$

or,



$$PSF = (PBBAR * ASEAL1) * PLSF$$

therefore,

$$d(PSF)/dPBBAR = ASEAL1(0) * PLSF(0)$$

(III-28)

and

$$d(PSF)/dZ = PBBAR(0) * d(PLSF * ASEAL1)/dZ$$

where

$$d(ASEAL1 * PLSF)/dZ = PLSF(0) * d(ASEAL1)/dZ + ASEAL1(0) * d(PLSF)/dZ$$

$$ASEAL1 = WIDTH * (LD - L3 * THETA) / SIN(31)$$

and

$$PLSF = L3 - (LD - L3 * THETA) / 2 * SIN(31)$$

therefore,

$$d(PLSF)/dZ = -1 / (2 * SIN(31))$$

and

$$d(ASEAL1)/dZ = WIDTH / SIN(31)$$

as a result,

$$d(PSF)/dZ = PBBAR(0) * ((PLSF(0) * WIDTH) / SIN(31) - ASEAL1(0) / (2 * SIN(31)))$$





(III-29)

$$d(psf)/d\theta = PBAR(0) * d(PLSF*ASEAL1)/d\theta$$

$$d(PLSF*ASEAL1)/d\theta = \\ PLSF(0) * d(ASEAL1)/d\theta + ASEAL1(0) * d(PLSF)/d\theta$$

$$d(ASEAL1)/d\theta = -L3*WIDTH/SIN(31)$$

$$d(PLSF)/d\theta = L3/(2*SIN(31))$$

as a result,

$$d(PSF)/d\theta = -PBAR(0) * ((PLSF(0) * L3*WIDTH)/SIN(31) - \\ ASEAL1(0) * L3/(2*SIN(31)))$$

(III-30)

For the after seal the derivatives proceed in the manner:

$$PSA = -(PBAR + PDIFF) * ASEAL2 * PLSA$$

$$d(PSA)/dPBAR = -ASEAL2(0) * PLSA(0)$$

(III-31)

In the same manner as shown in the previous paragraph, the partials of moment due to seals aft with respect to the other variables are:



$$d(psa)/dz = -PBBAR(0) * ((PLSA(0) * WIDTH) / SIN(32) + ASEAL2(0) / (2 * SIN(32)))$$

(III-32)

$$d(PSA)/dTHETA = -PBAR(0) * ((PLSA(0) * L4 * WIDTH) / SIN(32) + ASEAL2(0) * L4 / (2 * SIN(32)))$$

(III-33)

The partial of planing moment is:

$$PPLAN = L7 * HPLAN$$

(II-30)

$$d(PPLAN) = L7 * d(HPLAN)$$

$$d(PPLAN)/dTHETA = -L7 * KPLAN * V(0) ** 2$$

(III-34)

For this constant speed model all other partials are identically zero. The Plenum Gauge Pressure also has dynamic variation in the moment lever arm and the derivatives are:

$$PPRES = -XCP * HPRES$$

(II-34)

therefore,

$$dPPRES = -HPRES(0) d(XCP) - XCP(0) * d(HPRES)$$



where

$$XCP = XCP0 + (LDIFF * LD) / (2 * BUBHGT)$$

therefore,

$$d(XCP) / dZ = LDIFF / (2 * BUBHGT)$$

and

$$d(PPRES) = -((HPRES(0) * LDIFF) / (2 * BUBHGT)) * dZ \\ - XCP(0) * d(HPRES)$$

substituting equation (III-25) yields:

$$d(PPRES) = -((HPRES(0) * LDIFF) / (2 * BUBHGT)) * dZ \\ + XCP(0) * AB(0) * dPBBAR$$

Finally, the partial of pitch damping moment is:

$$PDAMP = -2 * PI * XLSS ** 2 * YLSS ** 2 * RHO * u * THETADOT / 8$$

(II-37)

let

$$2 * PI * XLSS ** 2 * YLSS ** 2 * RHO / 8 = KDAMP$$

then

$$PDAMP = -KDAMP * V * THETADOT$$

as a result, since this is a constant speed model,



$$d(PDAMP)/dTHETDT = -KDAMP * V(0)$$

(III-35)

where THETDT is the first derivative of theta with respect to time.

## 5. Total Differentials

Since  $F = M * A$  and the interest is in acceleration terms,  $A = F/M$  and  $dA = d(F)/M$ . the result being that all differentials must be divided by craft mass to achieve the desired linearized heave results. The same general comment holds true for moment differentials, that is; they all must be divided by the craft moment of inertia about the Y-axis.

All differentials must be sorted by the variable of differentiation so that the linearized differential equations of motion may be written. As an example;

$$dZDDOT = ((d(HBF)/dZ) + (d(HBA)/dZ)) / MASS * dZ +$$

The State Variable representation of the system will be presented in a later section of this chapter, however it is convenient to define the State Variables at this time.

$$\underline{X} = (dZ, dZDOT, dTHETA, dTHETDT, dMB)^t$$

(III-36)

and





$$\underline{y} = \underline{dPBBAR}$$

(III-37)

since PBBAR is not a state variable the definition for dPBBAR must be substituted for dPBBAR in all equations.

## 6. Total Heave Force Differentials in Z

Here the total differentials will be found. No explanatory text will be included unless special substitutions requiring an aside are necessary.

The buoyancy terms are:

$$DHBZ = d(HBF)/dZ + d(HBA)/dZ$$

(III-38)

A term must be included which was not previously derived to account for the sensitivity of seal forces due to PBBAR since it is not a State Variable.

$$DHSPZ = (d(HSF)/dPBBAR) * (d(PBBAR)/dZ)$$

as a result,

$$d(HSF)/dPBBAR = -K3*(LD(0) - L3*THETA(0))$$

(III-20)

and

$$DHSFPZ = -K3*(LD(0) - L3*THETA(0)) * d(PBBAR)/dZ$$

similarly for the after seal moment:



$$d(HSA)/dPBBAR = -K4*(LD(0) + L4*THETA(0))$$

(III-23)

and

$$DHSAPZ = -K4*(LD(0) + L4*THETA(0))*d(PBBAR)/dZ$$

The total seal force differential in Z becomes:

$$DHSZ = d(HSF)/dZ + d(HSA)/dZ + DHSFPZ + DHSAPZ$$

(III-39)

For plenum gauge pressure lift differential the same substitution must be made.

$$dHPRES = -AB(0)*dPBBAR$$

therefore,

$$DHPZ = -AB(0)*d(PBBAR)/dZ$$

(III-40)

Define the sensitivity coefficient of ZDDOT to Z to be:

$$DZZ = (DHBZ + DHSZ + DHPZ)/MASS$$

(III-41)

## 7. Total Differentials in THETA



The following partials are now defined:

$$DHBTH = d(HBF)/dTHETA + d(HBA)/dTHETA$$

(III-42)

$$DHSTH = d(HSF)/dTHETA + d(HBA)/dTHETA$$

(III-43)

$$DHPTH = d(HPLAN)/dTHETA$$

(III-44)

Define the sensitivity coefficient of ZDDOT due to THETA to be:

$$DZTH = (DHBTH + DHSTH + DHPTH)/MASS$$

(III-45)

### 3. Total Differentials due to MB

Here since dPBBAR is not a state variable the partial of PBBAR with respect to MB must be substituted in the plenum lift and seals lift terms.

$$d(HSF)/dPBBAR = -K3*(LD(0) - L3*THETA(0))$$

(III-20)

by substituting  $d(PBBAR)/dMB$  for  $dPBBAR$  the result is:



$$d(HSFP)/dMB = -K3*(LD(0) - L3*THETA(0)) * d(PBBAR)/dMB$$

(III-46)

similarly,

$$d(HSAP)/dMB = -K4*(LD(0) + L4*THETA(0)) * d(PBBAR)/dMB$$

(III-47)

$$d(HPRES) = -AB(0) * dPBBAR$$

using the previous substitution yields:

$$d(HPRES)/dMB = -AB(0) * d(PBBAR)/dMB$$

(III-48)

The sensitivity coefficient of heave due to MB can now be defined to be:

$$DZMB = (d(HSFP)/dMB + d(HSAP)/dMB + d(HPRES)/dMB) / MASS$$

(III-49)

## 9. Total Pitch Moment Differentials in Z

$$DPBZ = d(PBF)/dZ + d(PBA)/dZ$$

(III-50)

The dPBBAR substitution yields the partial of seals with respect to pressure with respect to Z.





$$DPSFPZ = ASEAL1(0) * PLSF(0) * d(PBBAR) / dz$$

(III-51)

$$DPSAPZ = -ASEAL2(0) * PLSA(0) * d(PBBAR) / dz$$

(III-52)

therefore the total seal differential in Z becomes:

$$DPSZ = DPSFPZ + DPSAPZ + d(PSF) / dz + d(PSA) / dz$$

(III-53)

Next the Plenum Gauge pressure term is derived:

$$d(PPRES) / dz = -HPRES(0) * LDIFF / (2 * BUBHGT)$$

$$d(PPRES) / dPBBAR = AB(0) * XCP(0)$$

therefore,

$$DPPRESZ = AB(0) * XCP(0) * d(PBBAR) / dz$$

and

$$DPRESZ = d(PPRES) / dz + d(DPPRESZ) / dz$$

(III-54)

The sensitivity coefficient of THETA with respect to Z becomes:

$$DTHZ = (DPBZ + DPSZ + DPPRES) / IYY$$



(III-55)

10. Total Moment Differentials in THETA

$$DPBTH = (d(PBF)/dTHETA + d(PBA)/dTHETA)$$

(III-56)

$$DPSTH = (d(PSF)/dTHETA + d(PSA)/dTHETA)$$

(III-57)

$$DPPLTH = d(PPLAN)/dTHETA$$

(III-58)

define the sensitivity coefficient for THETA to be:

$$DTHTH = (DPBTH + DPSTH + DPPLTH)/IYY$$

(III-59)

Also, define the sensitivity coefficient in THETDT to be:

$$DTHDTH = (d(PDAMP)/dTHDT)/IYY$$

(III-60)

11. Total Moment Differentials in MB



$$dPSAMB = (d(PSA)/dPBBAR) * (d(PBBAR)/dMB)$$

(III-61)

$$dPSFMB = (d(PSF)/dPBBAR) * (d(PBBAR)/dMB)$$

(III-62)

$$DPSMB = DPSAMB + DPSFMB$$

(III-63)

Inserting the dPBBAR substitution yields the partial of plenum gauge pressure moment with respect to MB:

$$d(PPRES)/dPBBAR = -XCP(0) * DHPRES$$

$$d(HPRES)/dMB = -AB(0) * d(PBBAR)/dMB$$

$$DPPMB = XCP(0) * AB(0) * d(PBBAR)/dMB$$

(III-64)

Define the sensitivity coefficient of THETA due to MB to be:

$$DTHMB = (DPSMB + DPPMB)/IYY$$

(III-65)

## 12. Total Differentials of Air Mass



$$d(MB)/dMB = (d(MB)/dPBBAR) * (d(PBBAR)/dMB)$$

define the sensitivity coefficient to be:

$$DMBMB = d(MB)/dMB$$

(III-66)

define the sensitivity coefficient in Z to be:

$$DMBZ = d(MB)/dZ$$

(III-67)

#### D. TOTAL MODEL SENSITIVITY COEFFICIENTS

##### 1. Heave Sensitivity Coefficients

The sensitivity coefficients for the linearized differential equation in heave acceleration are:

DZZ, DZTH, and DZMB.

##### 2. Pitch Sensitivity Coefficients

The sensitivity coefficients for the linearized differential equation in pitch acceleration are:

DTHZ, DTHH, DTHDTH, and DTHMB.

##### 3. Plenum Air Mass Sensitivity Coefficients





The sensitivity coefficients for plenum air mass are:

DMBZ, and DMBMB.

#### 4. Plenum Gauge Pressure Sensitivity Coefficients

The sensitivity coefficients for Plenum Gauge Pressure are:

DPBZ, and DPBMB.

#### E. STATE SPACE REPRESENTATION

The state matrix form of a dynamic system is:

$$\underline{\dot{X}} = A\underline{X} + B\underline{U}$$

where  $\underline{X}$  = a vector of variables representing the states of a set of first order linear differential equations, and  $\underline{U}$  is the vector of forcing functions, and A and B are the matrices of the respective gains. The form of the equations is:

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = X_3$$

$$\dot{X}_3 = K_1 X_1 + K_2 X_2 + K_3 X_3$$



for the linearized model of the CAB SES the state variables were defined to be :

$$\underline{x} = (dz, dzDOT, dTHETA, dTHETDT, dMB)^t$$

As a result the state matrix representation of the linearized unforced Heave-Pitch model is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ DZZ & 0 & DZTH & 0 & DZMB \\ 0 & 0 & 0 & 1 & 0 \\ DTHZ & 0 & DTHTH & DTHDTH & DTHMB \\ DMBZ & 0 & 0 & 0 & DMBMB \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

(III-68)

and

$$\dot{p}B\bar{B}AR = DPBZ \cdot x_1 + DPBMB \cdot x_5$$

III-69



#### IV. TIME DOMAIN VALIDATION

The linearized model developed in chapter III was validated against the the Six-degree-of-freedom nonlinear model. There were several changes made in the simulation techniques used, and at the same time the changes in center of pressure dynamic variation and planing forces were entered into the 6 DOF model. Establishment of equilibrium for the linear model was accomplished off-line using a TI-59 programmable hand-held calculator with printer.

The first change in simulation technique is the operating point at which the linear sensitivity coefficients were evaluated. In McIntyre's model these coefficients were evaluated at the initial operating point. In this work the coefficients are evaluated at the final steady-state equilibrium point since this is customary in linearization work.

The second major change in simulation was the method used to determine the planing force loss coefficient (PLCOEFF) and the establishment of the arbitrary location of the initial plenum center of pressure. In this model all heave forces are well defined (as modeled here) with the exception of planing force lift. As a result, craft weight and all heave forces except that due to planing lift are summed and the residual assumed to be due to the planing phenomenon. This residual is then used to determine (off-line) the value of PLCOEFF. Note that this method also accounts for any lift due to aerodynamic forces on the bow that were disregarded.



Next all moments are summed about the C.G. except the moment due to plenum pressure lift. The residual moment is assumed to be due to plenum pressure. This residual is used (again off-line) to establish the initial center of pressure. This accounts for any moments not previously accounted for (such as aerodynamic forces on the bow.) McIntyre used the planing lift coefficient to force the model into agreement with the 6 DOF model for natural pitch frequency and used a large, arbitrary, and static moment to force the models into steady state pitch angle agreement. It was felt that using the present method of determination would leave the equations in a form that would be more amenable to extraction of the factors most directly affecting craft characteristics as modeled herein.

#### A. LINEAR PROGRAM DESCRIPTION

User's instructions have been included as appendix A. The program flow can be isolated into four main sections.

1. First the values for craft weight, draft, pitch angle, plenum pressure and speed are entered. To initialize plenum air mass, draft is used to solve for initial plenum air volume, which is then used with initial plenum gauge pressure to solve equation II-15 for plenum air mass. Equation II-12 is then solved for  $Q_{in}$ .  $Q_{out}$  is set equal to  $Q_{in}$  and equation II-13 is solved for seal leakage area. All other heave and pitch nonlinear equations are solved and their values are listed along with residuals of heave and pitch summations. These residuals are added to the initial values of C.G. and angular acceleration and the initial





values are saved for the time domain simulation section.

2. The same operations are carried out for the steady state values of weight, draft, pitch angle, and plenum pressure. The residuals, forces and moments are again listed.

3. These steady state values are used to evaluate the sensitivity coefficients developed in chapter III which are also listed.

4. Finally the state variable solution is simulated using a four-point fixed step Runge-Kutta algorithm and listed for the print interval desired. The option is included to output either printer plot graphical display using the NPS printer plot subroutine PLOTP or the X-Y Versatec plot subroutine DRAWP. The latter should be used for final runs only, with PLOTP used on intermediate runs. Provisions are made for up to 300 discrete output points for the time domain tabular listing.

### B. SIX-DEGREE-OF-FREEDOM MODEL SOLUTION

Two changes were made in the 6 DOF model to account in a more analytic manner for the observed tendency of the XR-3 to pitch down as speed increased.

The first change was to remove an empirical equation used to correct the center of pressure [5] and replace it with the dynamic center of pressure variation developed in chapter II.

The second change was to include in the sidewall subroutine the nonlinear equation for planing forces, with the loss coefficient adjusted to make up (as



closely as possible) for lift loss due to the revised modeling of the plenum chamber.

Finally, the 6 DOF model was ran at both 6722 lbs and 6050 lbs weight to establish the initial and final operating points, followed by the weight removal transient run for time domain validation of the linear heave-pitch model.

It should be noted that further study of the 6 DOF model is needed to evaluate the results of the two changes and to more closely establish the arbitrary initial location of the center of pressure. The large amount of CPU and turn-around program times required to run the nonlinear model precluded including that study in this work. Also, for the nonlinear model, X-direction thrust was held constant during the run with the result that speed was 30.38 knots and still increasing at the end of the five second run. This effect will be further discussed in the next section.

### C. COMPARISON OF RESULTS

The graphical output for the five second runs are shown in figures 5 through 8 for the linear model and figures 9 through 12 for the nonlinear model. The fixed integration step size was 0.005 sec for the linear model and 0.001 sec for the nonlinear model.

#### 1. Plenum Gauge Pressure Transient

(see fig. 5 and 9)



In general both models exhibited a rapid initial reduction in pressure followed by a smooth transient back towards the final operating pressure. For the linear model the minimum occurred at 100 milli-seconds, and 110 milli-seconds for the 6 DOF model, with the respective pressures being 21.38 and 21.30. The final values were 24.31 and 24.08 for the linear and nonlinear model respectively.

As can be seen from the graphical data excellent agreement was achieved between the two models.

## 2. Draft Transient

(see fig. 6 and 10)

Both models exhibit a smooth exponential type transient towards the final, lower steady-state draft. The linear model transient settles towards a lower value (6.04 inches as compared to 6.44) which is in part attributed to a difference in modeling of the seal forces. The nonlinear model uses a complex combination of hydrodynamic and hydrostatic lift where the linear model uses a simple pressure differential effect. Also, since the nonlinear model is settling to a significantly lower pitch angle (see para. 4 below), the planing force effect is reduced in that model.

## 3. C.G Acceleration

(see fig. 7 and 11)

Both models show a rapid positive transient in acceleration that closely follows the negative transient



in plenum pressure. Both models reached the initial peak positive C.G. acceleration at approximately 20 milli-seconds after the plenum pressure completed the negative transient. The linear model appears to have greater coupling between pitch and heave looking at the C.G. acceleration, but this is not actually supportable since the linear model has greater pitch amplitude excursions about the exponential pitch transient to the lower steady state value (see fig. 8 and 12.) In general the two models show excellent agreement in transient and damped C.G. acceleration behavior.

#### 4. Pitch Transient

(see fig. 8 and 12)

It is in this area that the two models are in greatest disagreement. Both exhibit a general damped sinusoidal behavior about a center exponential transient to a lower steady state value. In both cases the initial excursion is downward. The natural pitch frequency of linear model is approximately 4.8 radians per second, whereas the nonlinear model exhibits a natural pitch frequency of 6.98 radians per second. There is felt to be one major factor contributing to this large discrepancy, which is the inclusion of the planing force in the nonlinear model. Data presented by Thaler and Gerba [14] showed a natural pitch frequency of 5.8 radians per second and much higher oscillations about the exponential transient. It is felt that the simplified planing forces of the linear model are included in the much more complex hydrodynamic equations of the nonlinear model and are therefore redundant in that model. Due to time constraints, that avenue of investigation is left for future studies. The linear





model is tending towards a steady state value on the order of 0.45 degrees whereas the nonlinear model is tending towards approximately 0.30 degrees. This large discrepancy is partially due to the redundant planing force in the nonlinear model and the fact that speed is also increasing in the nonlinear model which tends to accentuate the planing force and force the craft to pitch down.

In the next chapter a signal flow graph will be developed and Mason's Gain Rule applied to explain the linear model characteristic response in a more analytical fashion.



## V. SIGNAL FLOW GRAPH AND MASON'S GAIN RULE

### A. INTRODUCTION

The State Variables defined in Chapt II are used to develop a signal flow graph of the craft heave-pitch dynamics. Mason's Gain rule is applied to the signal flow graph to derive analytical S-domain polynomials expressing the transfer functions for pitch moment into pitch angle, heave acceleration into draft and the two cross gains. Using typical gains calculated in the time domain program developed in Chapt III the roots of these polynomials are located and it is shown that pitch dynamics may be very closely approximated by a second-order system. Heave dynamics are also simplified.

Using a simplified approach to the frequency response of the craft to non calm-water conditions, the basics are developed for later sea-state model work. A dynamic system modeling program available at the Naval Postgraduate School is used to generate Bode plots for the craft's frequency response in Heave and Pitch under the simplifying assumptions.

### B. SIGNAL FLOW GRAPH ANALYSIS

Fig 13 was generated using the sensitivity



coefficients defined in Chapter III. From that graph the following feed-forward and feedback loops and paths were identified:

1. Feed Forward,  $U(\text{THETA})$  into  $\text{THETA}$ .

$$P1 = (1/IYY) * (1/S^{**2})$$

2. Feed Forward,  $U(Z)$  into  $Z$ .

$$P2 = (1/MASS) * (1/S^{**2})$$

3. Feed Forward,  $U(\text{THETA})$  into  $Z$ .

$$P3 = (1/IYY) * DZTH * (1/S^{**4})$$

4. Feed Forward,  $U(Z)$  into  $\text{THETA}$ .

$$P4 = (1/MASS) * DTHZ * (1/S^{**4})$$

5. Feedback Loops.

$$L1 = DTHTH / S^{**2}$$

$$L2 = DTHDTH / S$$

$$L3 = (DZTH) * (DTHZ) / S^{**4}$$

$$L4 = (DZMB) * (DMBZ) / S^{**3}$$



$$L5 = DZZ/S^{**2}$$

$$L6 = DMBMB/S$$

$$L7 = (DZTH) * (DMBZ) * (DTHMB) / S^{**5}$$

### C. MASON'S GAIN RULE APPLIED

Mason's gain rule for a closed multi-loop system is:

$$T = \text{SUM}(P_k * \text{DELTA}_k) / \text{DELTA}$$

where,

T = transmission gain from an input to an output node.

DELTA = the graph determinant

DELTA<sub>k</sub> = cofactor of the kth path

P<sub>k</sub> = gain of the direct path from input to output

Using Mason's gain rule the graph determinant (DELTA) is found to be of the form:

$$D(s) = (B5*s^{**5} + B4*s^{**4} + B3*s^{**3} + B2*s^{**2} + B1*s + B0) / s^{**5}$$

The forward direct path U(THETA) to THETA is:

$$N1(s) = (1/IYY) * (1/s^{**5}) * (A3*s^{**3} + A2*s^{**2} + A1*s + A0)$$

The forward path U(Z) to Z was found to be:





$$N2(s) = (1/MASS) * (1/S^{**5}) * (A3*s^{**3} + A2*s^{**2} + A1*s + A0)$$

The forward direct path U(THETA) to Z is:

$$N3(S) = (1/IYY) * DZTH * (S + A0) / S^{**4}$$

The forward path for draft coupling into pitch (U(Z) to THETA is:

$$N4 = (1/MASS) * DTHZ * (S + A0) / S^{**4}$$

#### D. SOLUTION OF THE INDIVIDUAL POLYNOMIALS

The gains used below were taken from a typical time domain run as developed in chapter IV.

DZZ = -2178.0  
 DZTH = -121.3  
 DZMB = -5499.0  
 DTHZ = 28.78  
 DTHTH = -22.31  
 DTHMB = 72.1  
 DMBZ = -28.79  
 DMBMB = -73.11

#### 1. Solution of the Graph Determinant

Inserting the numerical values for the typical sensitivity coefficients listed above and using only the



dominant terms where possible, the roots of the graph determinant were found to be:

$$\Delta = (S^2 + 71.8S + 2066)(S + 0.5086)(S^2 + 0.72S + 22.74)/S^5$$

These factors represent a pair of complex poles located at  $-35.71 \pm j27.84$  , a real pole located at  $-0.5085$  , a pair of complex conjugate roots located at  $-0.360 \pm j 4.755$  , and a fifth order zero located at the origin.

## 2. Solution of $N_1(S)$

$$N_1(S) = (1/IYY)(S + 0.427)(S^2 + 72.68S + 2146)/S^5$$

This polynomial factors into a real zero located at  $-0.427$  , a pair of complex conjugate zeros located at  $-36.34 \pm j 28.74$  , and a fifth order pole located at the origin.

## 3. Solution of $N_2(S)$

$$N_2(S) = (1/MASS)(S^2 + 0.7721S + 22.35)(S + 72.38)/S^5$$

Which factors into a pair of complex conjugate zeros located at  $-0.3856 \pm j 4.712$  , a real zero at  $-72.38$  , and a fifth order pole located at the origin.



#### 4. Solution of N3(S)

$$N3(S) = (1/IYY) * DZTH * (S + 73.11) / S^{**4}$$

Which represents a real zero at -73.11, and a fourth order pole at the origin.

#### 5. Solution of N4(S)

$$N4(S) = (1/MASS) * DTHZ * (S + 73.11) / S^{**4}$$

Which has the same pole and zero locations as N3(S).

### E. SIMPLIFICATION OF TRANSFER FUNCTIONS

#### 1. T1(S)

$$T1(S) = THETA(S) / U(THETA, S)$$

Therefore from Mason's Gain Rule:

$$T1(S) = N1(S) / DELTA$$

$$T1(S) = (1/IYY) * (S + 35.34 \pm 428.7) * (S + 0.427) / \\ (S + 35.1 \pm j 27.84) * (S + 0.508) * (S + 0.36 \pm j 4.755)$$



To simplify let the complex conjugate zeros at  $-36.34 \pm j28.7$  cancel the poles at  $-35.9 \pm j27.84$ , and the real zero at  $-0.427$  cancel the pole at  $-0.508$ . The result is the simplified second order system:

$$T1(S) = (1/IYY) / (S^2 + 0.72s + 22.74)$$

Inspection of the time domain pitch response with respect to natural frequency and damping confirms this basic underdamped second order response.

## 2. T2(S)

$$T2(S) = Z(S) / U(Z, S)$$

$$T2(S) = (1/MASS) * (S + 0.3856 \pm j4.712) * (S + 72.38) / (S^2 + 35.1 \pm j 27.84) * (S + 0.5085) * (S + 0.35 \pm j4.755)$$

Canceling the pole at  $-0.36 \pm j 4.755$  with the zero at  $-0.3856 \pm j4.712$  results in the simplified response:

$$T2(S) = (1/MASS) * (S + 72.33) / (S + 0.508) * (S^2 + 71.8S + 2066)$$

Further simplification can be made if  $(S + 72.33) / (S + 0.508)$  is approximated by  $72.33 / (S + 0.508)$  (which is valid for longer time period considerations.) As a result  $T2(S)$  becomes:





$$T_2(S) = (72.33/MASS)/(S+0.508)*(S^2 + 71.8*S + 2066)$$

which has poles located at  $-0.508$ , and  $-35.9 \pm j27.83$ . This is in quite close agreement with the results obtained by Thaler and Gerba 14 for the same airflow condition ( $K_g = 1.0$ ) for their heave-only linearized model. Their results were a real pole at  $-0.33$  and a complex pair at  $-35.1 \pm j34.4$ . This is of the form:

$$T(S) = K/(S + 0.38)*(S^2 + 70.2*S + 2415)$$

### 3. T3(S)

$$T_3(S) = Z(S)/U(\Theta)$$

Here it is noted that there are no pole zero pairs that conveniently cancel so that the transfer function becomes:

$$T_3(S) = (1/IYY)*DZTH*S*(S + 72.33) / (S + 35.1 \pm j 27.84)*(S + 0.5085)*(S + 0.36 \pm j4.775)$$

Making the simplification used in the previous section, the result is:

$$T_3(S) = (72.33/IYY)*DZTH*S/(S^2 + 71.8*S + 2066)*(S + 0.508)*(S^2 + 0.72*S + 22.74)$$

Which can be further simplified by looking at the long term response vice initial few milli-seconds to  $(S/(S + 0.508)) = 2$  so that:



$$T3(S) = (142/IYY)*DZTH/(s**2 + 71.8*s + 2066)*(s**2 + 0.72*s + 22.74)$$

#### 4. T4(S)

Since the transfer fundtions are the same except for the gains the solutions are the same except for the gains:

$$T4(S) = (1/MASS)*DTHZ * S*(S + 72.33)/DELTA$$

where the same simplifying assumptions may be applied.

### F. SIMPLIFIED BODE PLOT ANALYSIS

#### 1. Simplifying Assumption

(see fig. 12)

The assumption is made that the craft linearized frequency response transfer functions can be separated into two transfer functions.

The first block  $G1(W,U,...)$  expresses the disturbing moments and accelerations generated by the craft interacting with the waves. This function was developed by Thaler and Gerba [14] for the heave-only model. In that simpler model the function  $G1$  was a complex function of encounter frequency, angle of incidence, craft length compared to encounter wavelength etc.. This portion of the overall frequency response of this model is left for future work due to time



constraints.

The second block  $G_2(s)$  represents the heave-pitch dynamics of the model as developed in chapters II, III, and IV, which respond to the disturbing moments and accelerations from block  $G_1$ .

The Transfer functions  $T_1$  and  $T_2$  (unsimplified) were simulated using the IBM-360 simulation language DSL with  $s = j\omega$  with the following results.

## 2. Bode Plots for Pitch Dynamics

(see figs. 15 and 16 )

Figure 15 represents the normalized (numerator gain constants set to 1) gain in dB's versus  $\omega$ . Note that the abscissa scale is  $\omega$  in powers of 10 since this is a convenient means to achieve a logarithmic scale on the X-Y plotter output. As can be seen the response strongly supports the simplification to second-order system. These plots represent the full transfer function  $T_1(j\omega)$  vice the simplified system. Figure 16 further supports the simplification as the phase plot starts at zero degrees and shows only a very slight positive increase prior to the rapid change toward  $-180$  characteristic of an underdamped system. Also note that above the damped natural frequency the magnitude plot falls at  $-12$  dB per octave.

## 3. Bode Plots for Heave Dynamics

(see figs. 17 and 18)



Figure 17 represents the normalized magnitude versus  $\Omega$  for the heave dynamics. The system is characteristic of an overdamped third order system with small gain increases at the craft's natural pitch frequency, which represents the pitch dynamics feeding back into heave dynamics. However the gain magnitude curve has already fallen 20 dB below its lower frequency response gain before the pitch coupling begins to manifest itself. As can be seen in fig. 18 the phase plot shows the same third order pole response with slight second order zero influence at the natural pitch frequency.





## VI. CONCLUSIONS AND RECOMMENDATIONS

As shown in Chapters IV and V the simplified Pitch-Heave model developed herein is a very reasonable simulation of the XR-3 test craft for the given operating conditions. Heave, Pressure, and C.G. Acceleration were in very close agreement. Pitch on the other hand showed some difference in natural frequency and damping factor. It is felt that this difference is primarily attributable to the inclusion of a redundant planing force calculation in the 6 DOF model. Historically the results of the 6 DOF computations<sup>14</sup> for pitch response have been very similar to the results for the linear model, in both natural frequency and damping characteristics. This can be attributed to the fact that the additional planing force did not exist in the original 6 DOF program.

Since one of the major concerns with the 6 DOF model is the excessive computational times involved in sea-state simulations the conclusion may be drawn that eventual substitution of the sidewall and seal force linear model equation in the 6 DOF program will substantially reduce CPU time, with some resultant loss in computational accuracy. By way of comparison, disregarding compiling time and output time, the linear model required approximately 9 seconds of CPU time for the 5 second simulation while the 6 DOF model required 9 minutes of CPU for the 5 second run. This is a computation time improvement on the order of 60 times.

From the time-domain plenum gauge pressure data it can



be seen that the dynamic variation in effective plenum roof area has a noticeable effect on the plenum pressure transient behavior as compared to previous results 14 . The initial pressure down transient was faster, dropped to a lower value , and began it's recovery more quickly. Therefore the plenum roof dynamic variations strongly affects the C.G. acceleration characteristics of the craft and therefore it's transient behavior.

The dynamic center of pressure variation developed herein proved to be a very draft sensitive phenomenon. Although the length of the other lever arms as modeled herein change by a similar amount, percentage of variation is radically different. For the seals the variation is on the order of inches in 10 feet, where the variation in lever arm for plenum pressure lift is on the order of inches in a single foot (or less.) This implies that dynamic C.P. variation is the dominant factor in steady state pitch angle versus steady state draft.

Planing forces as included in this model are now in a form where the basic factors as far as craft construction characteristics are readily recognized. This force provides part of the answer to craft pitch-down characteristics as initially investigated by Reidel [9], with the remainder of the moment being developed by the C.P. variation for pressure. The planing force provides a clue as to how, during construction, the craft's pitch amplitude excursion characteristics may be controlled.

Several areas for future investigation have become apparent in this work.

1. Verification of the effect of dynamic center of



pressure variation and planing forces as incorporated in the 6 DOF model over the entire speed operating range.

2. Incorporate the simple modeling for sidewalls and seals into the 6 DOF model and investigate the trade-off between CPU time and computational accuracy.

3. A closer look at the seal forces used herein. It is possible that a simple gain factor needs to be included in these equations in order to optimize the steady-state seal forces with respect to actual load data as measured during experimental test runs of the craft 10 .

4. Incorporate sea state forces and moments into the simplified model by simulation of the forces and moments that are generated by craft interaction with the sea. Evaluate the accuracy of the method used by comparison with data generated by the simple model and the 6 DOF model in sea-state conditions.

5. Analytical investigation of the craft physical characteristics that have predominant influence on the natural pitch frequency and damping by using the gain terms from the results of the signal flow graph analysis done in Chapter V.

6. Use the simplified model to develop a ride-comfort control system. Test the resulting design on the 6 DOF model, and compare the results obtained with those suggested by ROHR [7]. It has been known empirically for some time that the craft exhibited simple second order behavior in the pitch mode. Now that this fact has been analytically verified the implications for an automatic control system for pitch response are powerful. A simple control system can be visualized



that does not involve either complex optimal control system design work or hardware. A simple acceleration and rate feedback control system is implied, with the possibility of adding a speed input to the system to compensate for the localized linearity of the craft characteristics.





## APPENDIX A

### LINEAR SES MODEL USER'S GUIDE

#### A. General Comments

The program as listed following Appendix B is intended for useage via the hot card reader at NPS.

It is intended to run in Fortran G with Plotting option (PROC = FORTCLGP.)

The program is structured on the assumption that the initial and final state variables are in equilibrium prior to data entry, but will run regardless. The force and moment residual listings will indicate the value of imbalance, if any.

The craft physical dimensions are alterable by changing the cards in the craft dimension section of the main program, which are keyed to the craft geometry herein.

#### B. Data Deck Setup.

The /\* and //GO.SYSIN DD \* cards are inserted following the end card in Subroutine VERSAP, followed by the data deck as described

1. Plot option card (IPLOT), format I1. IPLOT = 1 =  
VERSATEC X-Y plotter output.

IPLOT = 0 = PRT PLOT output



## 2. List option card (ILIST), Format I1

ILIST = 1 = Forces and moments, with residuals and all sensitivity coefficients listed.

ILIST = 0 = Input craft operating conditions only listed.

## 3. Time and Integration variables.

TI, TF, NPTS, NSTEP Format (2F10.4, 2I10)

Ti = problem start time (normally 0.0)

TF = Problem stop time (seconds)

NPTS = number of tabular output points desired.

NSTEP = number of integration steps.

note: it is best to make NSTEP an integer multiple of NPTS, and never less than NPTS.

## 4. Planing coefficient and arbitrary center of pressure input, i.e. READ( ) AKP, PLCOEFF

Format = 2F10.4

## 5. Initial Conditions card.

The initial craft conditions are read in in the following order: weight (Lbs.), Draft (in), Pitch angle (Deg.), PBBAR (psf), and speed (fps.) Format is 5F10.4

## 6. Final Conditions.

The final conditions are read in the following order and format: weight (Lbs), Draft (in), Pitch angle (Deg), and PBBAR (psf), Format 4F10.4. Since this is a constant speed model, initial and final speed are assumed equal.

## 7. Label Cards.

Label cards for VERSATEC output are the last 10 cards.



There are five graphs in this mode. . Draft(in) , C.G. Acceleration (G's) , Pitch Angle (Deg.) , Pitch Acceleration (Deg/SEC\*\*2) , and PBBAR (Each graph requires two label cards, Format (6A8) , which print on two lines below the graph. The graphs are output in the order listed above.



## APPENDIX B

### DRAWINGS AND FIGURES





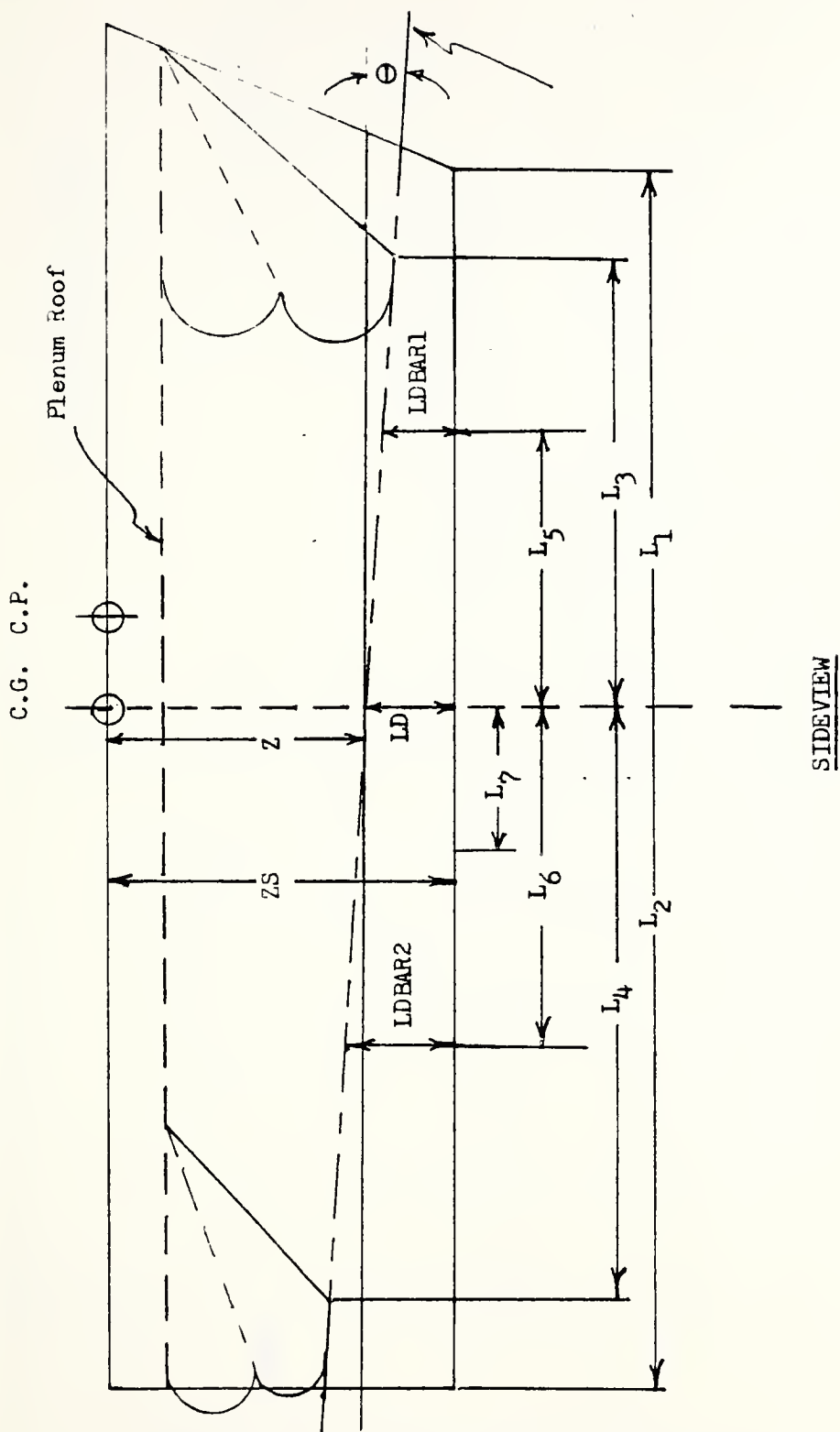


Figure 1 - ASSUMED GEOMETRY



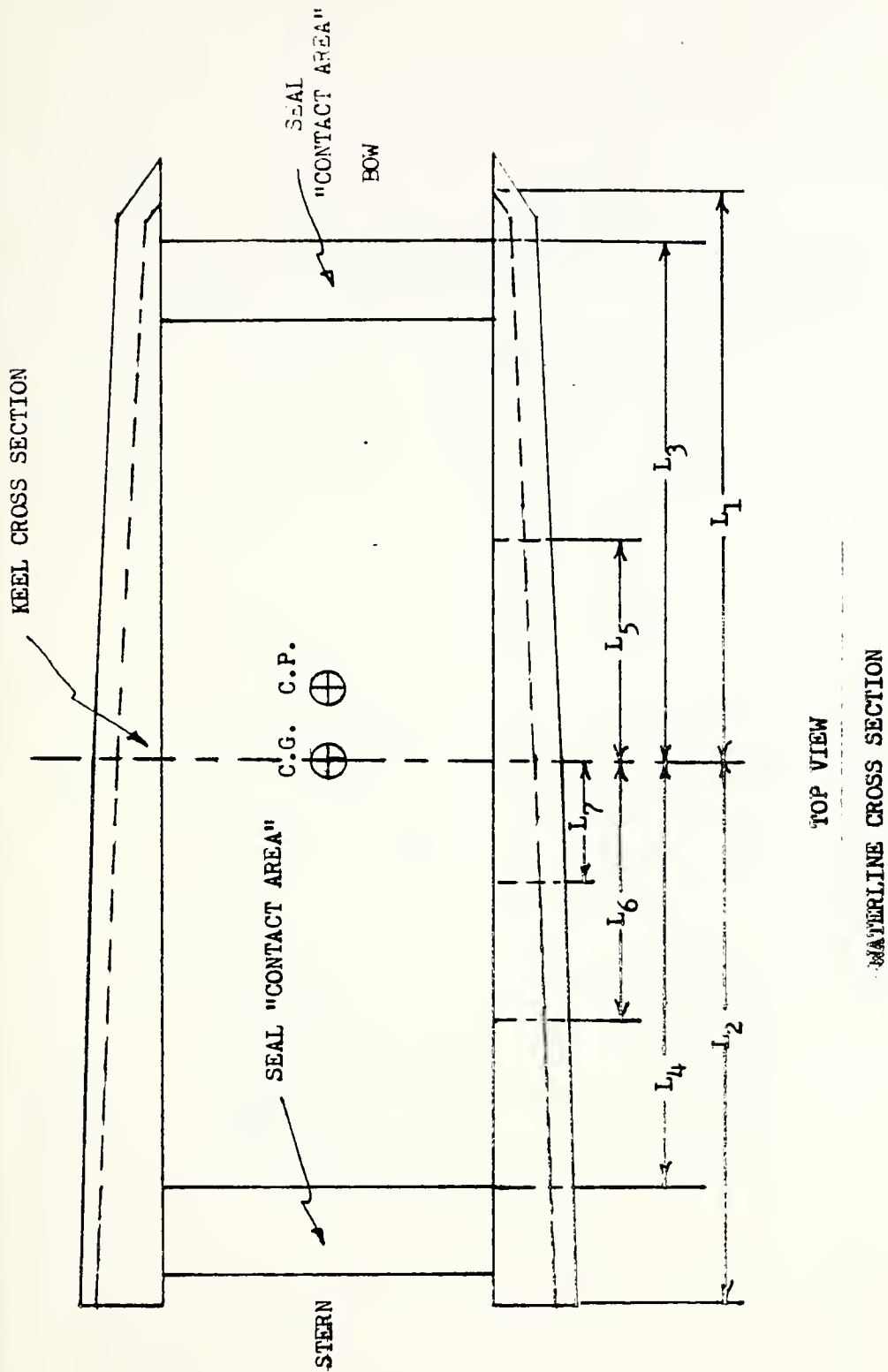
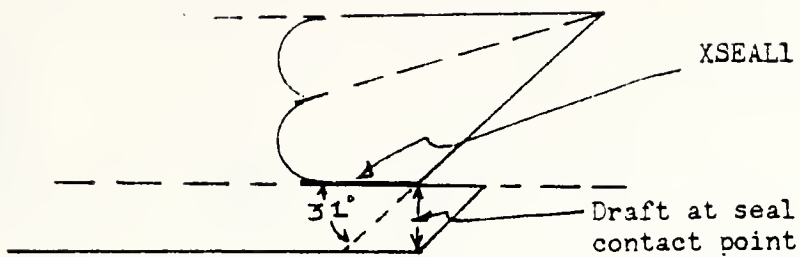


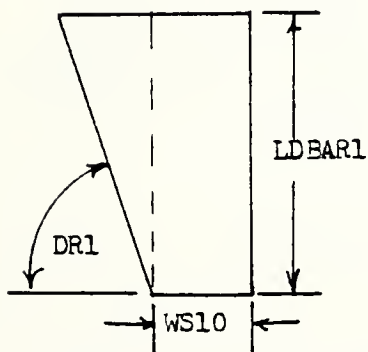
Figure 2 - ASSUMED GEOMETRY (CONT)





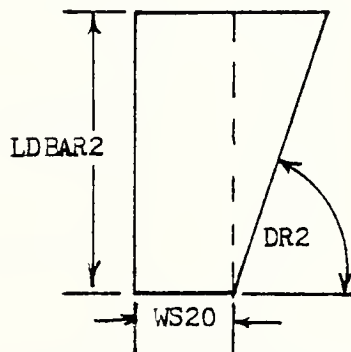
BOW SEAL  
(Details of Stern Seal the same)

FIGURE 3A



VERTICAL CROSS SECTION  
FORWARD

FIGURE 3B



VERTICAL CROSS SECTION  
AFT

FIGURE 3C

Figure 3 - ASSUMED GEOMETRY (CONT)



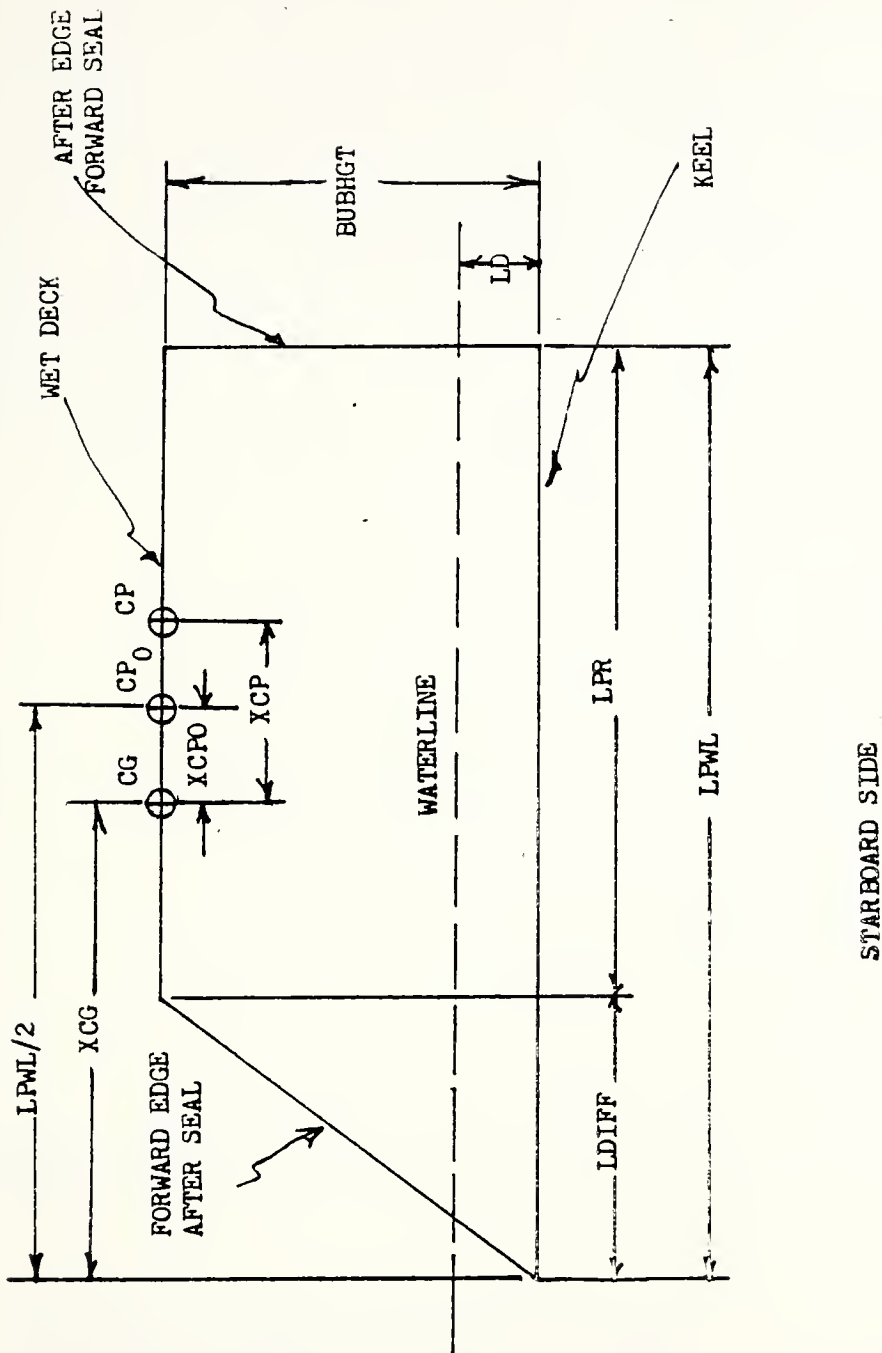
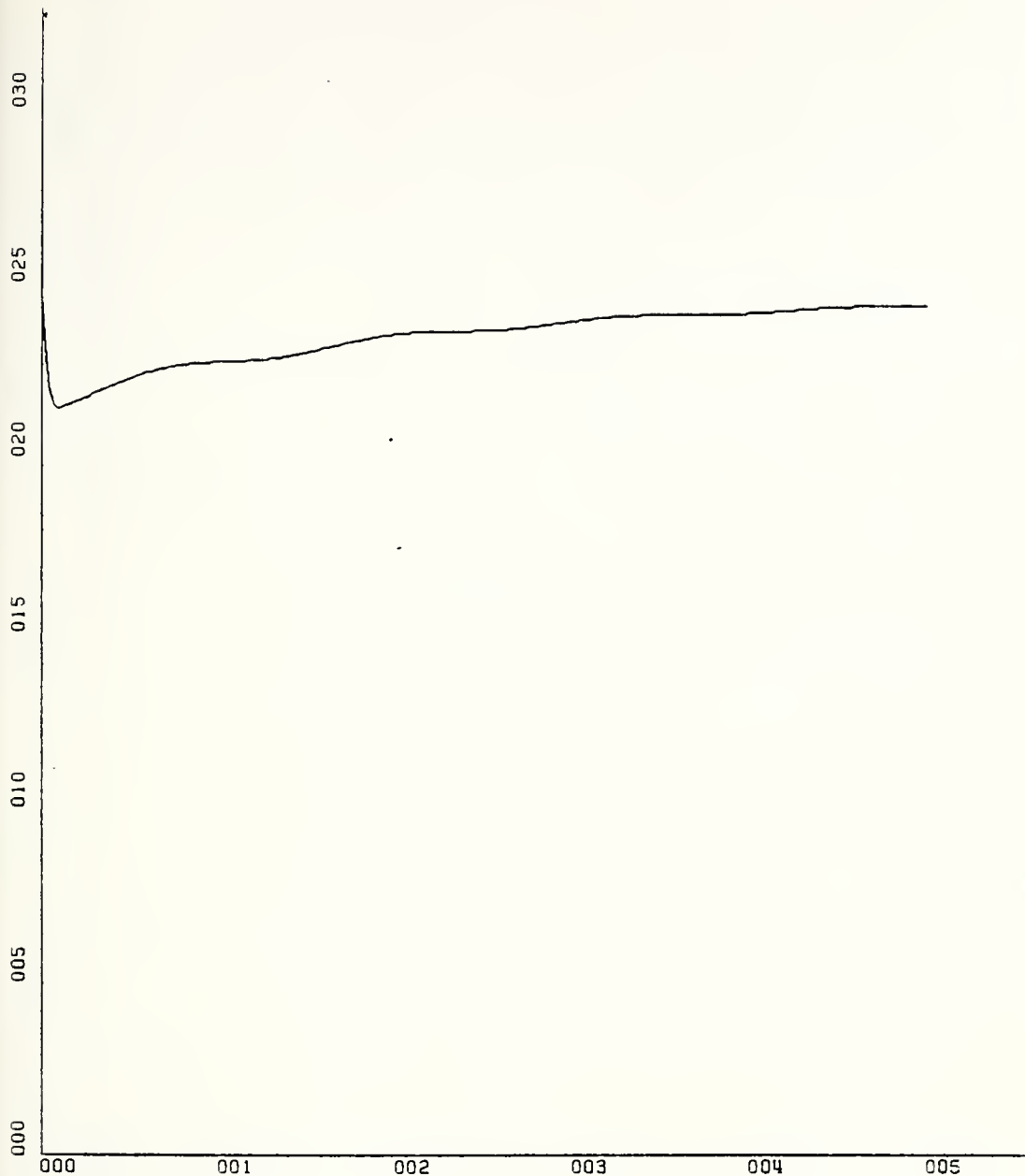


Figure 4 - PLENUM CHAMBER MODELING







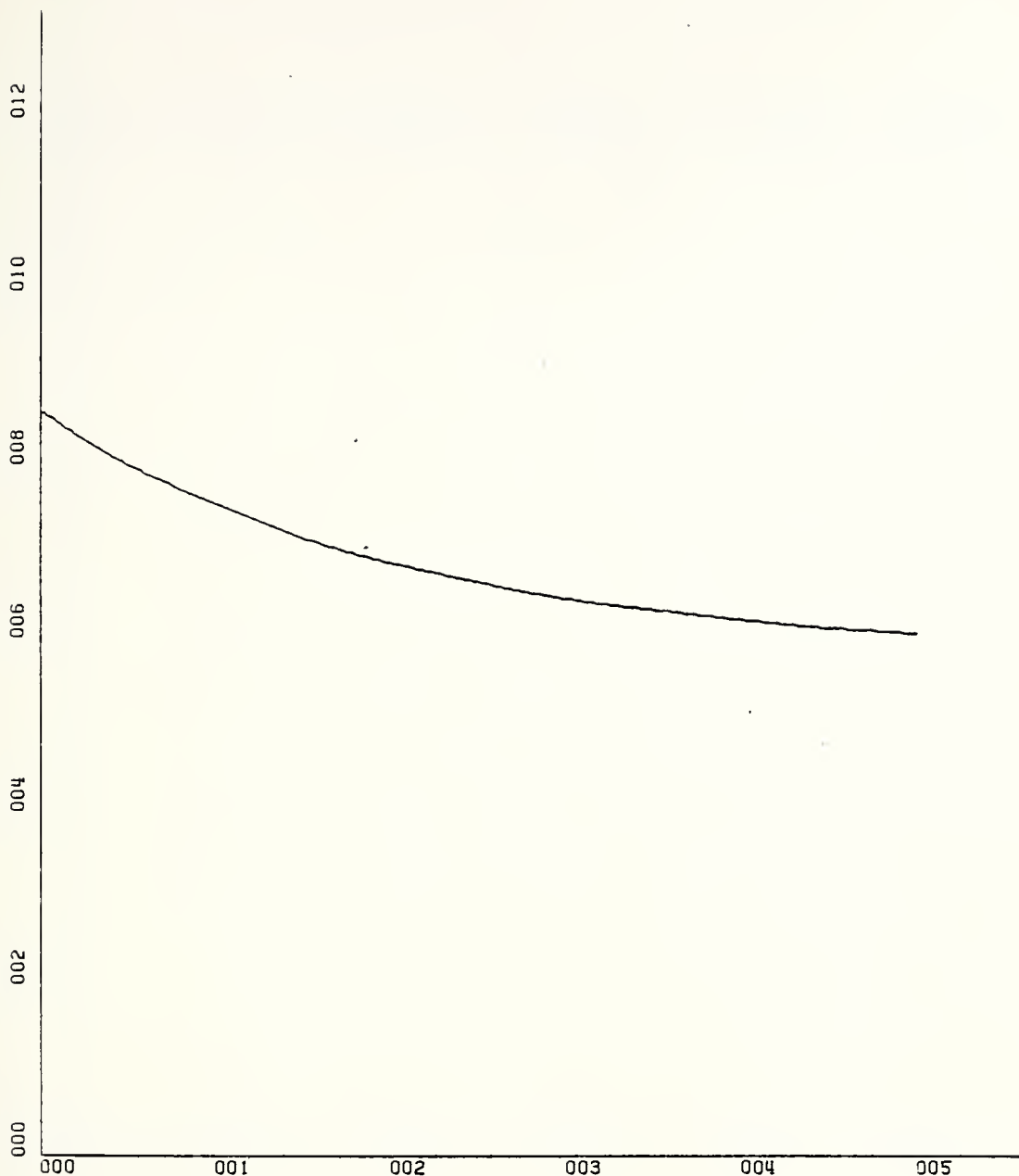
X-SCALE=1.00E+00 UNITS INCH.

Y-SCALE=5.00E+00 UNITS INCH.

PLOT IS PLENUM GAUGE PRESS  
CAB SES LINEAR 2 DOF MODEL

Figure 5 - LINEAR PLENUM PRESSURE TRANSIENT





X-SCALE=1.00E+00 UNITS INCH.

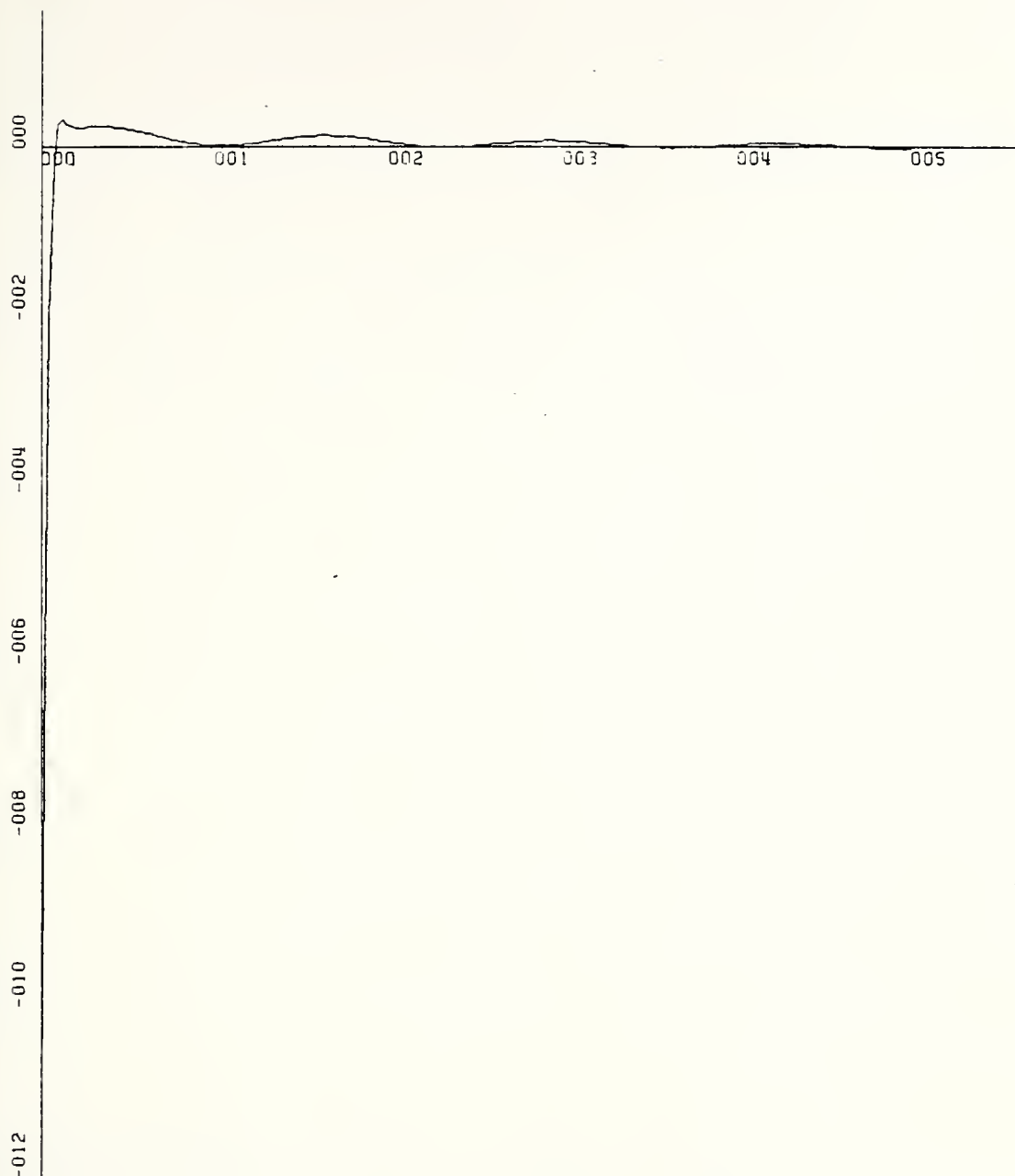
Y-SCALE=2.00E+00 UNITS INCH.

PLOT IS DRAFT (INCHES)

CAB SES LINEAR 2 DOF MODEL

Figure 6 - LINEAR DRAFT TRANSIENT





X-SCALE=1.00E+00 UNITS INCH.

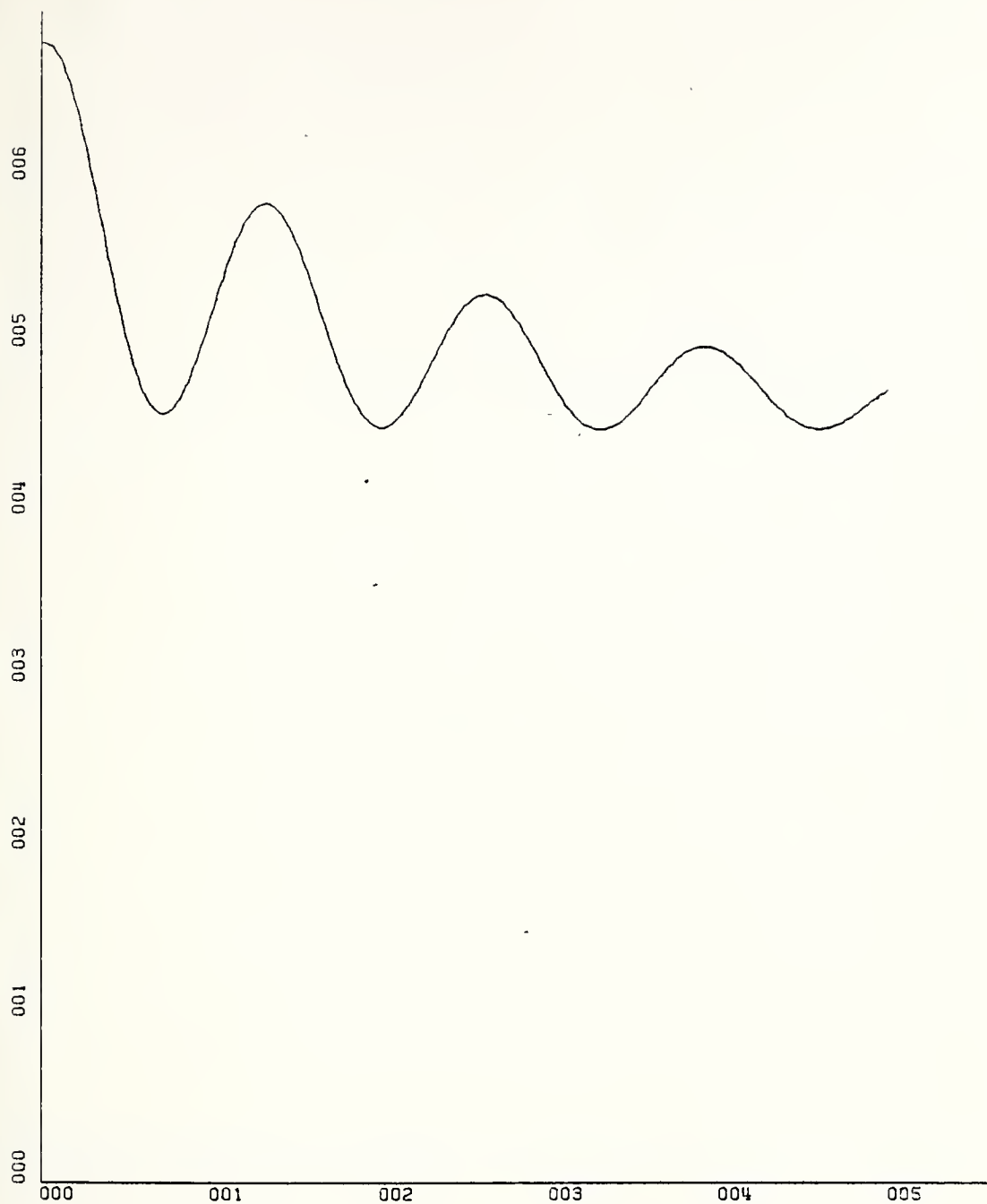
Y-SCALE=2.00E-02 UNITS INCH.

PLOT IS CG ACCEL (GEES)

CAB SES LINEAR 2 DOF MODEL

Figure 7 - LINEAR C.G. ACCELERATION





X-SCALE=1.00E+00 UNITS INCH.

Y-SCALE=1.00E-01 UNITS INCH.

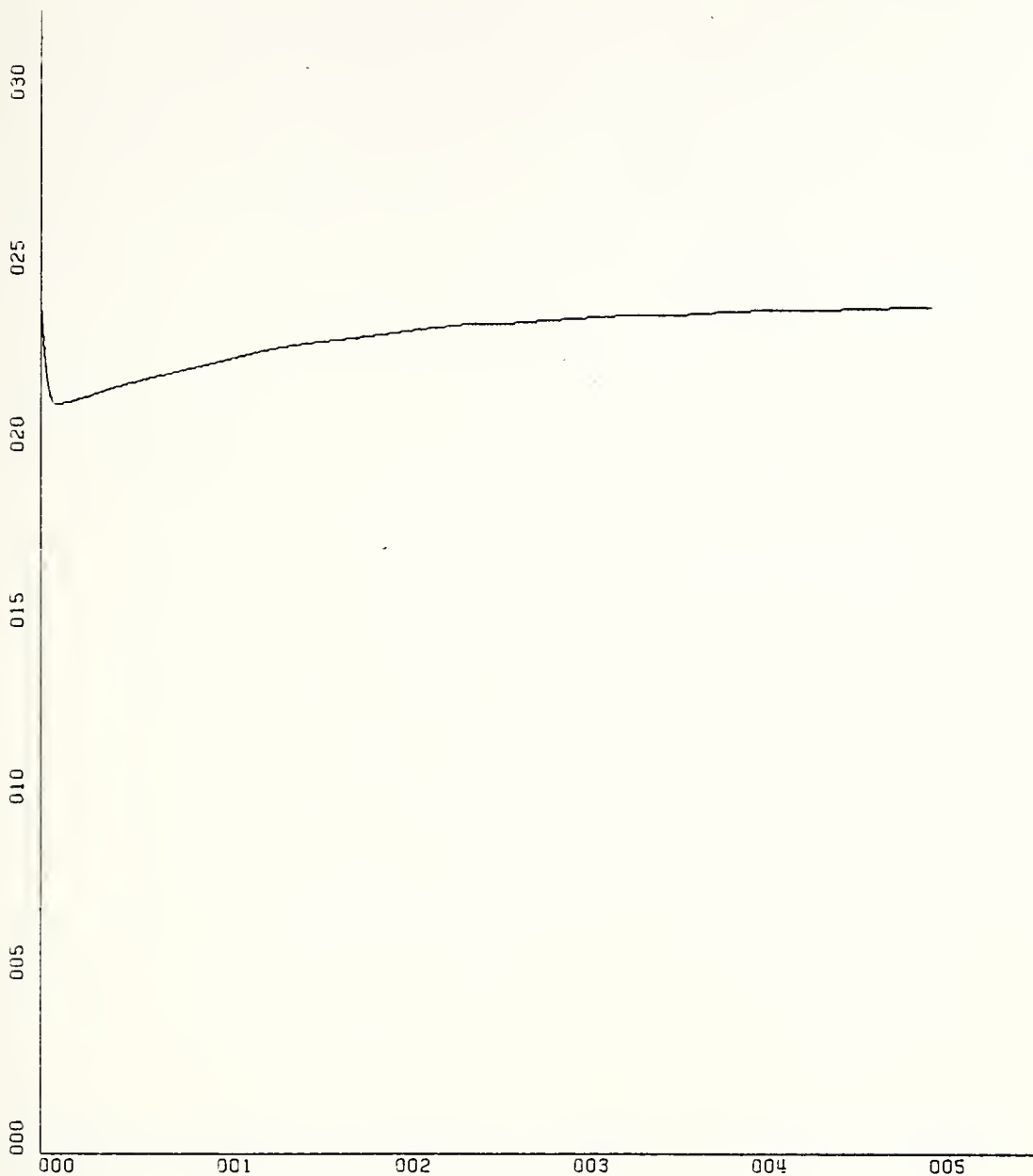
PLOT IS PITCH ANGLE

CAB SES LINEAR 2 DOF MODEL

Figure 8 - LINEAR PITCH TRANSIENT







X-SCALE=1.00E+00 UNITS INCH.

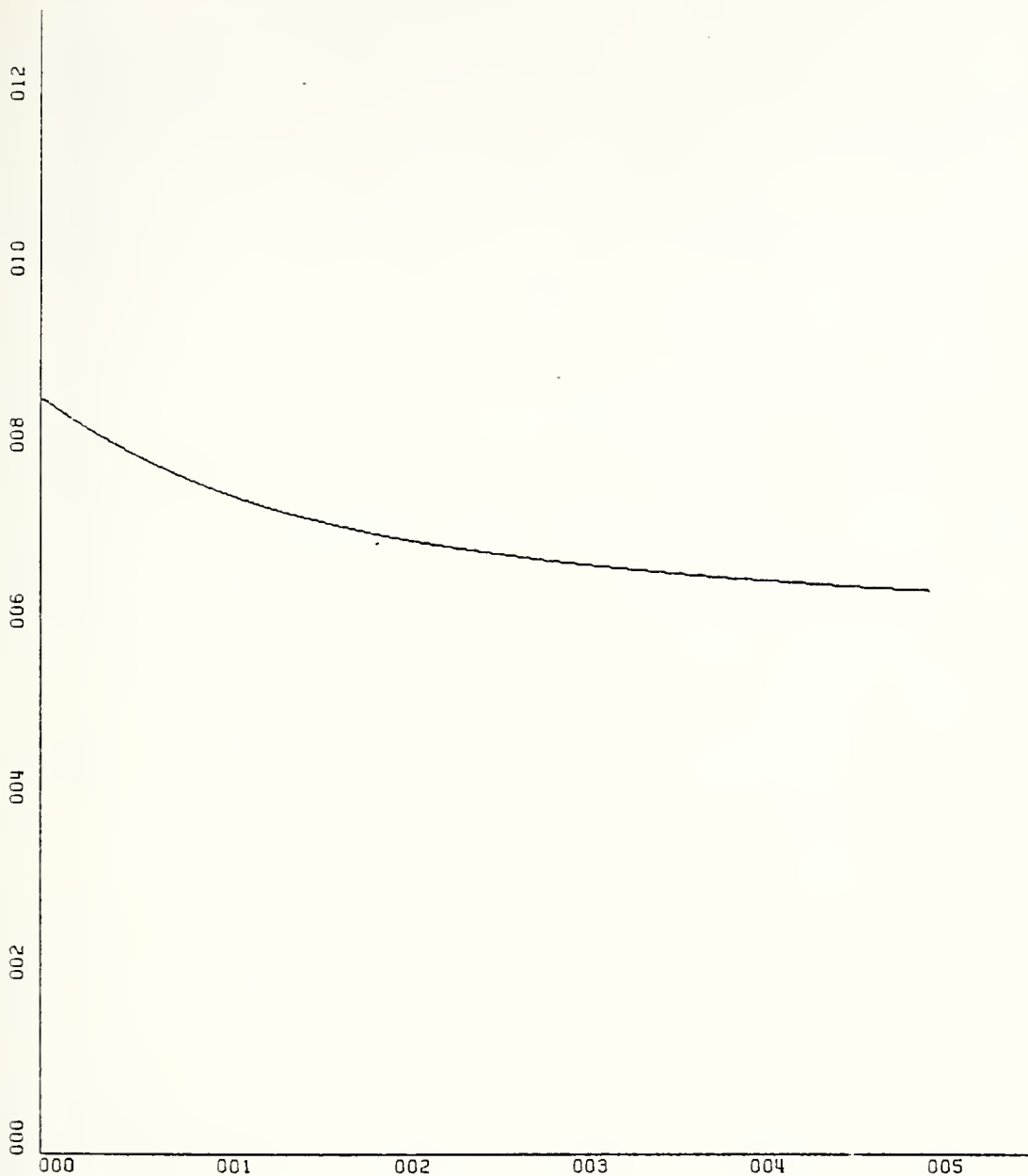
Y-SCALE=5.00E+00 UNITS INCH.

30 KNOT RUN

PLOT IS PLENUM PRESSURE

Figure 9 - 6 DOF PLENUM PRESSURE TRANSIENT





X-SCALE=1.00E+00 UNITS INCH.

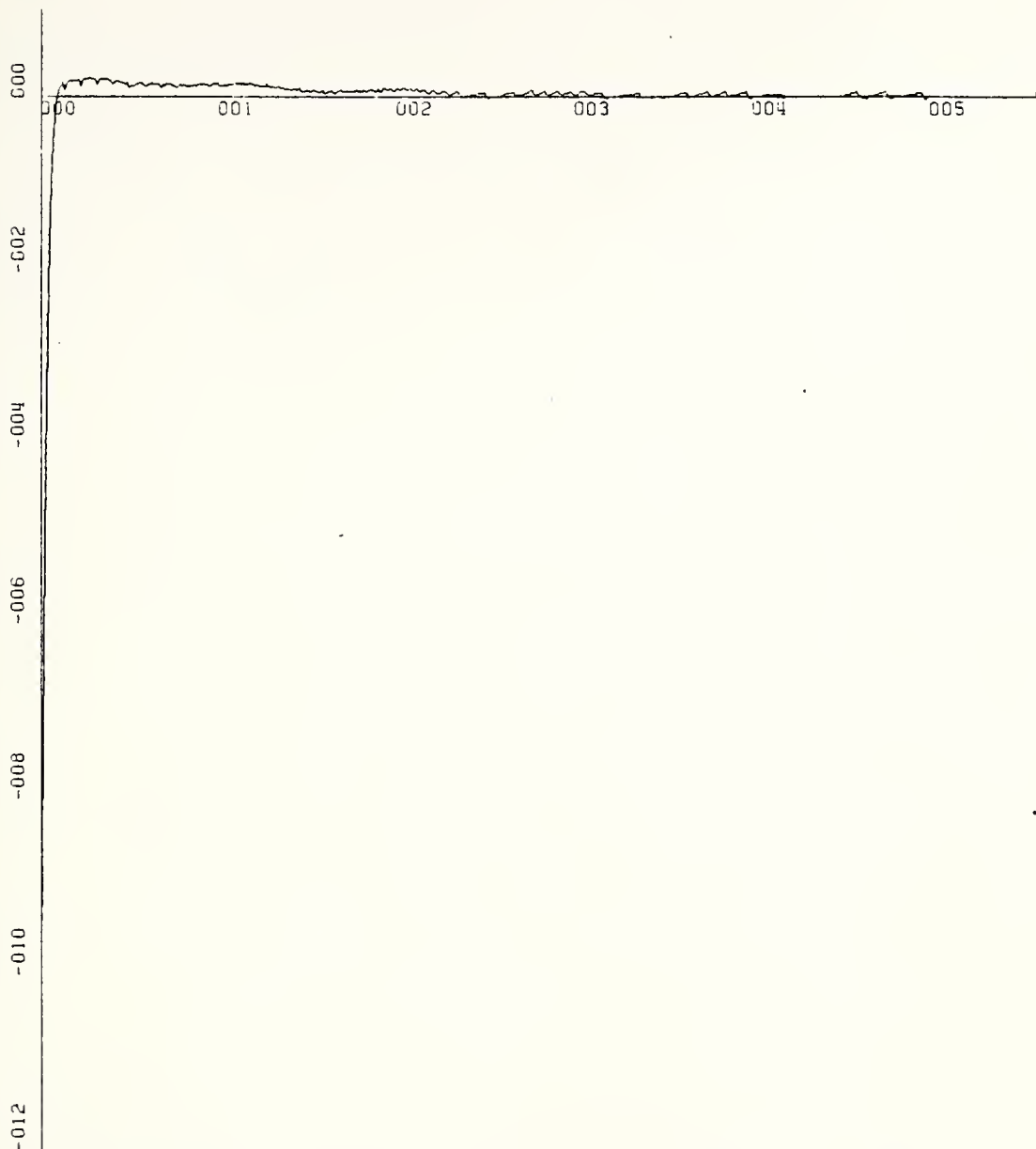
Y-SCALE=2.00E+00 UNITS INCH.

30 KNOT RUN

PLOT IS Z DISPLACEMENT

Figure 10 - 6 DOF DRAFT TRANSIENT





X-SCALE=1.00E+00 UNITS INCH.

Y-SCALE=2.00E-02 UNITS INCH.

30 KNOT RUN

PLOT IS C.G. ACCELERATION

Figure 11 - 6 DOF C.G. ACCELERATION





X-SCALE=1.00E+00 UNITS INCH.  
Y-SCALE=1.00E-01 UNITS INCH.  
30 KNOT RUN  
PLOT IS PITCH ANGLE

Figure 12 - 6 DOF PITCH TRANSIENT









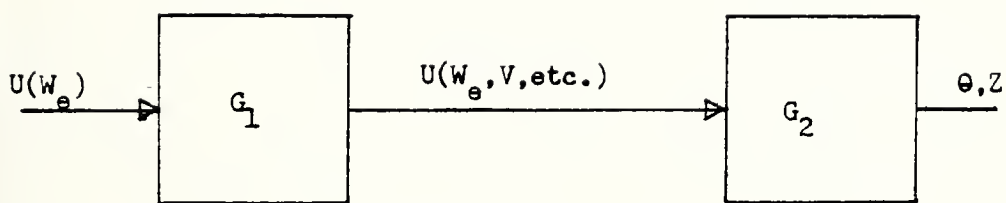
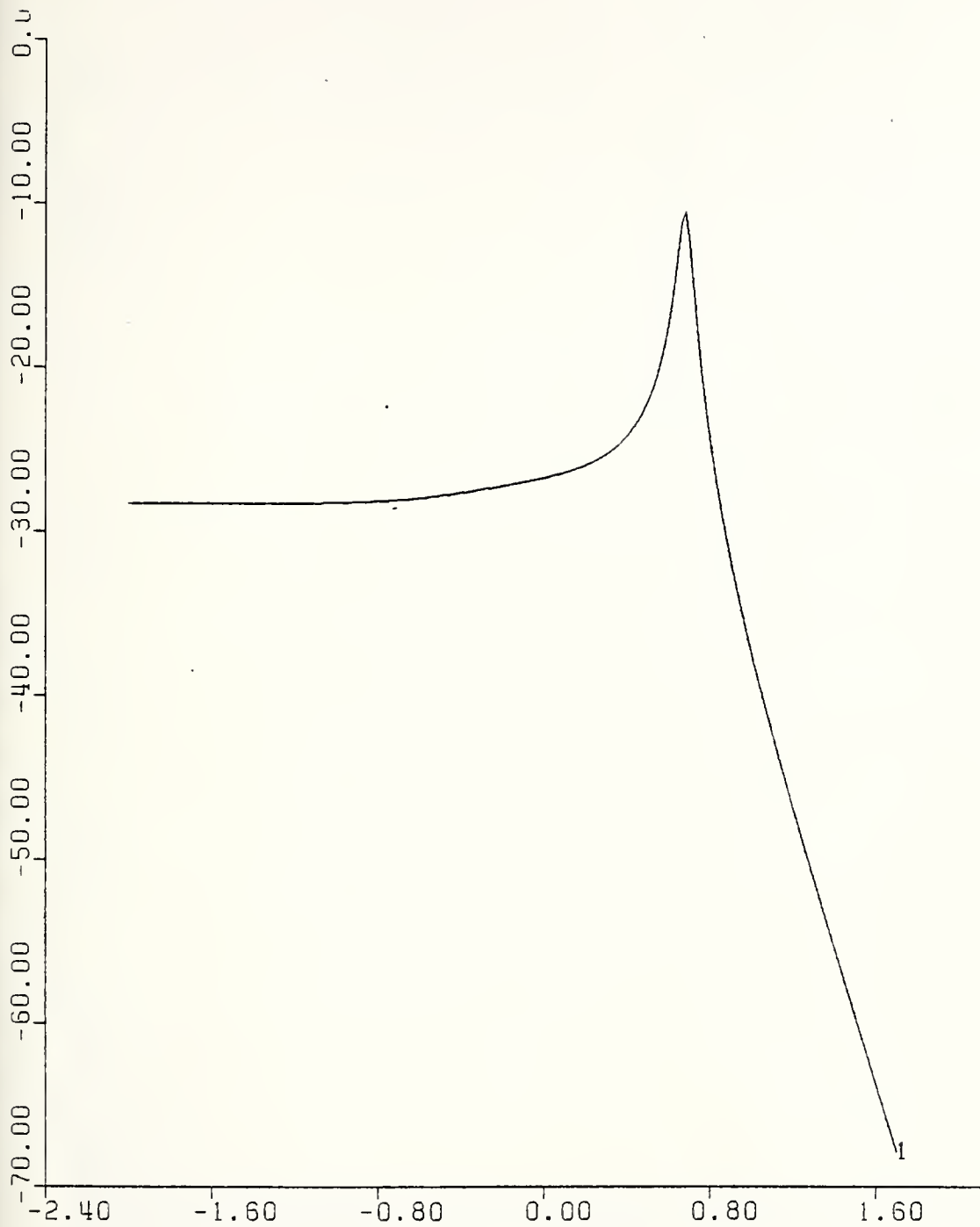


Figure 14 - HEAVE PITCH TRANSFER FUNCTION

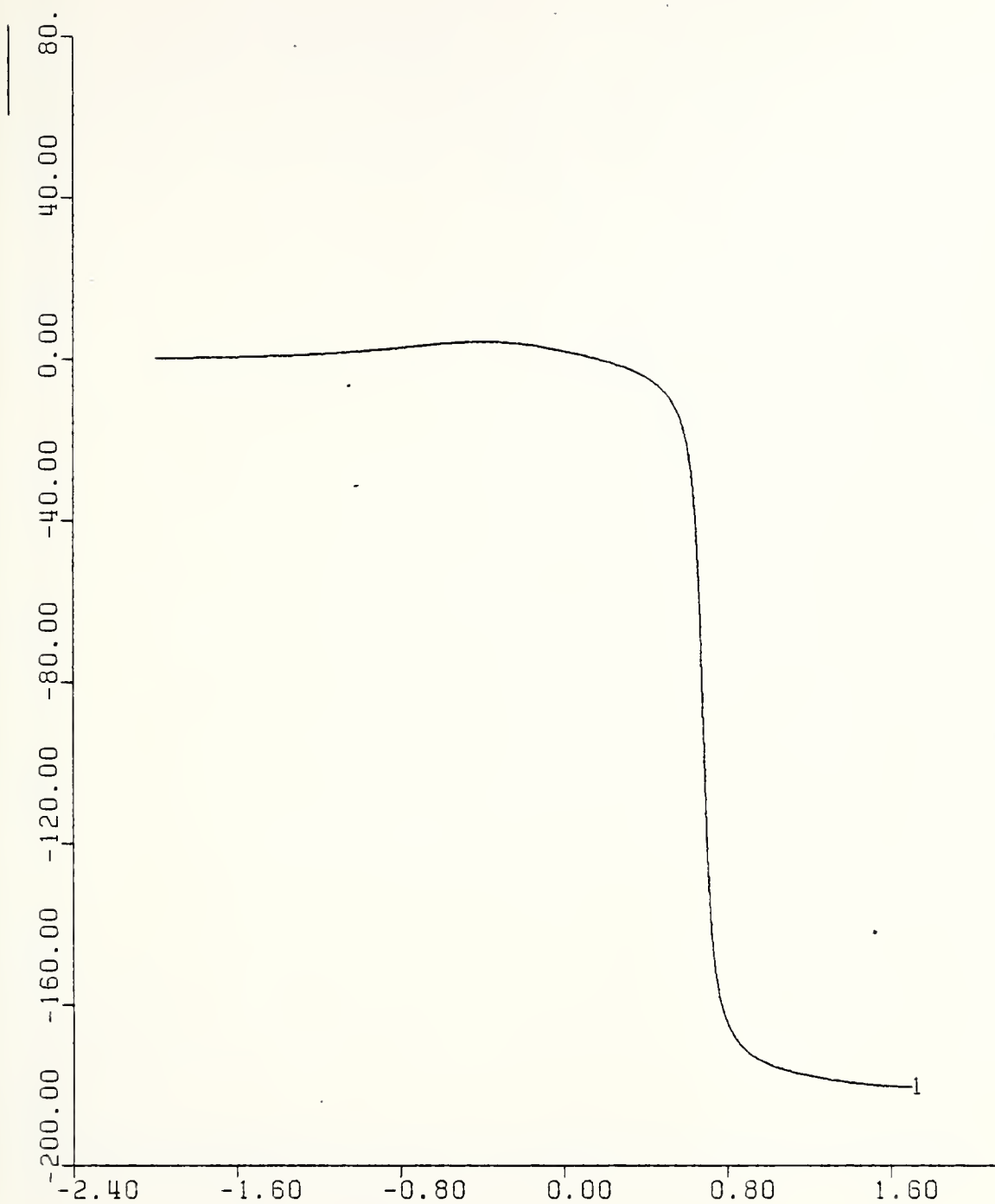




XSCALE= 0.80      UNITS/INCH    RUN   NO. 1  
YSCALE= 10.00     UNITS/INCH    PLOT NO. 1

Figure 15 - LINEAR PITCH BODE PLOT (MAG)



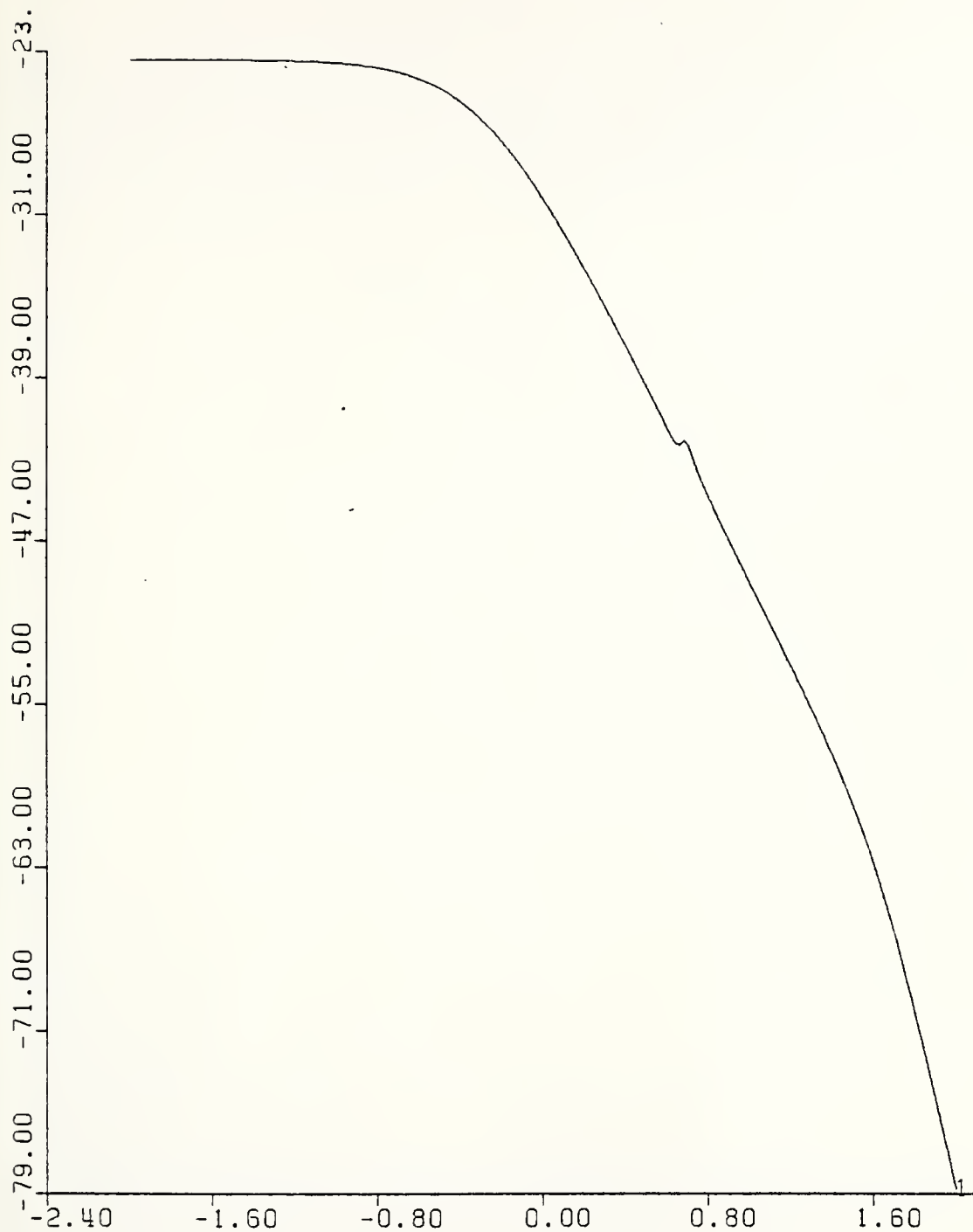


XSCALE= 0.80      UNITS/INCH   RUN   NO. 1  
YSCALE= 40.00    UNITS/INCH   PLOT NO. 2

FIGURE 10 - LINEAR PITCH BODE PLOT (PHASE)



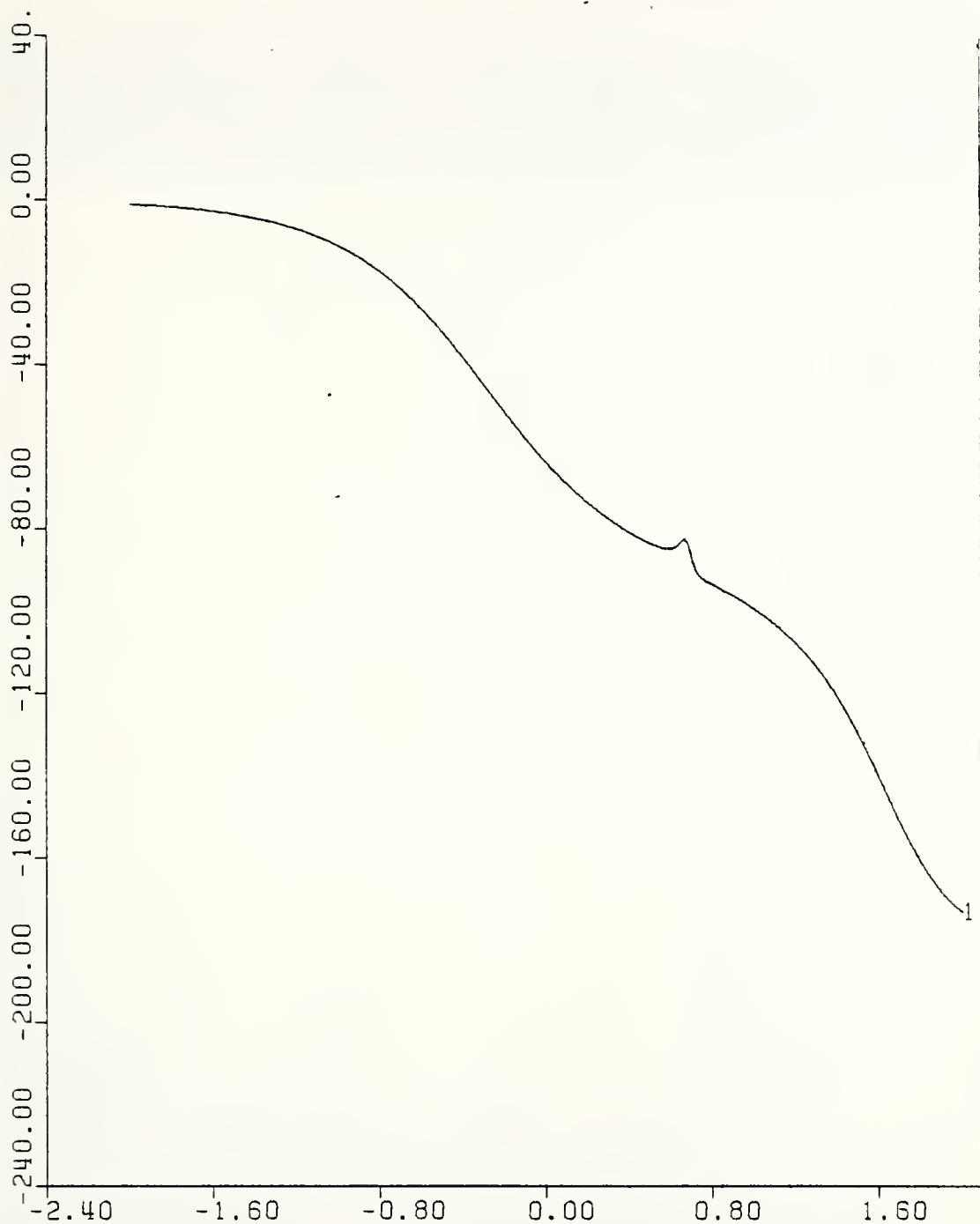




XSCALE= 0.80                      UNITS/INCH    RUN    NO. 1  
YSCALE= 8.00                      UNITS/INCH    PLOT NO. 1

Figure 17 - LINEAR HEAVE BODE PLOT (MAG)





XSCALE= 0.80            UNITS/INCH    RUN   NO. 1  
YSCALE= 40.00          UNITS/INCH    PLOT NO. 2

Figure 18 - LINEAR HEAVE BODE PLOT (PHASE)



```

DIMENSION OUFVEC(300,6), PLCTV(300), TIME(300), QPOINT(6)
DIMENSION ZI(2), THETA1(2), AMBI(2), PBBARI(2)
COMMON/INTEG/X(6), XDOT(6), T,H,NPASS
COMMON/DERIV/DZ2, DZPH, DZMB, DTHZ, DTHH, DTHMB, DTHDTH, DMBMB, DMBZ,
1DPBDZ, DPBMB, ACCEL, PACCEL
DATA OUFVEC, PLCTV, TIME/1800*0.0, 300*0.0, 300*0.0/
WRITE(6,9600)
WRITE(6,9605)

PHYSICAL CONSTANTS

G=32.2
GAMMA=1.4
PA=2110.0
PI=3.141593
RHO=2.0
RHOA=0.002378

CRAFT DIMENSIONS

WIDTH=10.0
AL1=10.3
AL2=9.96
AL3=11.2
AL4=8.26
AL5=AL1/2.0
AL6=AL2/2.0
AL7=4.227
AL8=17.1
DR1=61.0
DR2=68.5
WS10=0.2
WS20=0.52
CN=0.9
ALPR=15.72
ALPWL=19.65
ALDIFF=ALPWL-ALPR
ALDIFF=ALDIFF*WIDTH
XCPO=ALPWL/2.0-(119.6/12.0)
ABW=ALPWL*WIDTH
ABR=ALPR*WIDTH
BUBHGT=1.915
VN=((ALPR+ALPWL)/2.0)*BUBHGT*WIDTH
ZS=2.5
QIO=35.0
EN=5.0
A33S=0.7088
AIYY=9320

```

CCC

CCC

SES00010  
SES00020  
SES00030  
SES00040  
SES00050  
SES00060  
SES00070  
SES00080  
SES00090  
SES00100  
SES00110  
SES00120  
SES00130  
SES00140  
SES00150  
SES00160  
SES00170  
SES00180  
SES00190  
SES00200  
SES00210  
SES00220  
SES00230  
SES00240  
SES00250  
SES00260  
SES00270  
SES00280  
SES00290  
SES00300  
SES00310  
SES00320  
SES00330  
SES00340  
SES00350  
SES00360  
SES00370  
SES00380  
SES00390  
SES00400  
SES00410  
SES00420  
SES00430  
SES00440  
SES00450  
SES00460  
SES00470  
SES00490









SES00950  
 SES00960  
 SES00970  
 SES00980  
 SES00990  
 SES01000  
 SES01010  
 SES01020  
 SES01030  
 SES01040  
 SES01050  
 SES01060  
 SES01070  
 SES01080  
 SES01090  
 SES01100  
 SES01110  
 SES01120  
 SES01130  
 SES01140  
 SES01150  
 SES01160  
 SES01170  
 SES01180  
 SES01190  
 SES01200  
 SES01210  
 SES01220  
 SES01230  
 SES01240  
 SES01250  
 SES01260  
 SES01270  
 SES01280  
 SES01290  
 SES01300  
 SES01310  
 SES01320  
 SES01330  
 SES01340  
 SES01350  
 SES01360  
 SES01370  
 SES01380  
 SES01390  
 SES01400  
 SES01410  
 SES01420

ASEAL1=WIDTHH\*XSEAL1  
 ASEAL2=WIDTHH\*XSEAL2  
 WSPL1=WS10\*AL1  
 WSPL2=WS20\*AL2  
 XCP=XCPO+(ALDIFF\*ALD/BUHGHGT)/2.0+XCPC  
 AB=ABW-(ABW-ABR)\*(ALD/BUHGHGT)  
 VB=VN-AB\*ALD  
 PB=PA+PBBAR  
 AMB=RHOA\*VB/(PB/PA)\*\*(GAMMA)  
 QIN=EN\*(QIO-PBBAR)  
 AL=QIN/(CN\*SQRT(2.0\*PBBAR/RHOA))  
 ZI(J)=ALD  
 THETA(J)=THETA  
 AMBI(J)=AMB  
 PBBARI(J)=PBBAR

C C C

CALCULATE FORCES AND MOMENTS

HPRES=-AB\*PBBAR  
 HBF=-AK1\*ALD1\*WS1  
 HBA=-AK2\*ALD2\*WS2  
 HSF=-PBBAR\*ASEAL1  
 HSA=-((PBBAR+2.0)\*ASEAL2  
 AKP1=-RHO\*THETA\*AKP\*V\*V\*PI\*2.0  
 AKP2=-RHO\*THETA\*AKP\*V\*V\*PI\*2.0  
 HPP=AKP1\*WSPL1  
 HPA=AKP2\*WSPL2  
 PPRES=-XCP\*HPRES  
 PBF=-AL5\*HBF  
 PBA=AL6\*HBA  
 PPLAN=AL7\*(HPF+HPA)  
 PLSPF=AL3-XSEAL1/2.0  
 PLSA=AL4+XSEAL2/2.0  
 PSF=-HSF\*PLSP  
 PSA=HSA\*PLSA  
 IF(IGE.1) GO TO 10  
 RES1=W+HBF+HBA+HSF+HSA+HPF+HPA+HPRES  
 RES2=PBF+PBA+PSF+PSA+PPLAN+PPRES  
 ACCEL=RES1/AMASS  
 PACCEL=RES2/ALVY  
 IF(ILLIST.NE.1) GO TO 6

C C C

LIST FORCES, MOMENTS, AND RESIDUALS

WRITE(6,9620) W,HBF,HBA,HSF,HSA,HPF,HPA,HPRES,RES1  
 WRITE(6,9625) PBF,PBA,PSF,PSA,PPLAN,PPRES,RES2

C C

INPUT FINAL OPERATING POINT



```

C      6 I=1
      7 J=2
      8 READ(5,9510) WF,ALD,THETA,PBBAR
      9 WRITE(6,9615) WF,ALD,THETA,PBBAR,V
      10 AMASS=WF/G
      11 WSTEP=W-WF
      12 GO TO 5

C      10 OUTPUT FORCES, MOMENTS, AND RESIDUALS AT FINAL OPER POINT

      11 RES1=WF+HBF+HBA+HSF+HSA+HPF+HPA+HPRES
      12 RES2=PBF+PBA+PSF+PSA+PPLAN+PPRES
      13 IF(ILIST.NE.1) GO TO 11
      14 WRITE(6,9620) WF,HBF,HBA,HSF,HSA,HPF,HPA,HPRES,RES1
      15 WRITE(6,9625) PBF,PBA,PSF,PSA,PPLAN,PPRES,RES2

C      11 INITIALIZE STATE VECTORS

      12 QPOINT(1)=ZI(1)*12.0
      13 QPOINT(2)=ACCEL/G
      14 QPOINT(3)=THETA(1)*RADEG
      15 QPOINT(4)=PACCEL
      16 QPOINT(5)=AMBI(1)/RHOA
      17 QPOINT(6)=PBBARI(1)
      18 ALD0=ZI(1)*12.
      19 THETA0=THETA(1)
      20 AMB0=AMBI(1)
      21 PBBAR0=PBBARI(1)

C      22 COMPUTE SENSITIVITY COEFFICIENTS

      23 SIN31=SIN(31.0*DEGRAD)
      24 SIN32=SIN(32.0*DEGRAD)

C      25 DERIVATIVES OF PBBAR AND MB

      26 DPBDZ=GAMMA*(AB-2.0*ABDIFF*ALD/BUBHGT)*PB/VB
      27 DPBMB=GAMMA*PB/AMB
      28 DMBPB=-RHOA*(EN+CN*AL/(RHOA*SQRT(2.0*PBBAR/RHOA)))
      29 DMBZ=DMBPB*DPBDZ
      30 DMBMB=DMBPB*DPBMB

C      31 DERIVATIVES OF HEAVE FORCES W/RESPECT TO Z

      32 DHBFPZ=-AK1*(((ALD-AL5*THETA)/TAN(DR1)+WS10)
      33 DHBAPZ=-AK2*(((ALD+AL6*THETA)/TAN(DR2)+WS20)
      34 DHBZ=(DHBFPZ+DHBAPZ)

```

```

SES01430
SES01440
SES01450
SES01460
SES01470
SES01480
SES01490
SES01500
SES01510
SES01520
SES01530
SES01540
SES01550
SES01560
SES01570
SES01580
SES01590
SES01600
SES01610
SES01620
SES01630
SES01640
SES01650
SES01660
SES01670
SES01680
SES01690
SES01700
SES01710
SES01720
SES01730
SES01740
SES01750
SES01760
SES01770
SES01780
SES01790
SES01810
SES01820
SES01830
SES01840
SES01850
SES01860
SES01870
SES01880
SES01890
SES01900

```



DHSFZ=-AK3\*PBBAR-AK3\*(ALD-AL3\*THETA)\*DPBDZ  
 DHSAZ=-AK4\*(PBBAR+2.)\*-AK4\*(ALD+AL4\*THETA)\*DPBDZ  
 DHSZ=DHSFZ+DHSZ  
 DHPBZ=-AB\*DPBDZ  
 DZZ=(DHBZ+DHSZ+DHPBZ)/AMASS

C  
C  
C

DERIVATIVES OF HEAVE FORCES W/RESPECT TO THETA

DHBPTH=AL1\*AL5\*RRHO\*G\*(((ALD-AL5\*THETA)/TAN(DR1)+WS10)  
 DHBATH=-AL2\*AL5\*RRHO\*G\*(((ALD-AL5\*THETA)/TAN(DR2)+WS20)  
 DHSFTH=AK3\*AL3\*PBBAR  
 DHSATH=-AK4\*AL4\*(PBBAR+2.0)  
 DHPFTH=-RHO\*AKP\*V\*V\*WSPL1\*PI  
 DHPATH=-RHO\*AKP\*V\*V\*WSPL2\*PI  
 DHBTH=DHBPTH+DHBATH  
 DHSFTH=DHSFTH+DHSATH  
 DHPFTH=DHPFTH+DHPATH  
 DZTH=(DHBTH+DHSFTH+DHPFTH)/AMASS

DERIVATIVE OF HEAVE FORCE W/RESPECT TO MB

C  
C  
C

DHPBMB=-AB\*DP3MB  
 DHSFMB=-ASEAL1\*DPBMB  
 DHSAMB=-ASEAL2\*DPBMB  
 DZMB=(DHPBMB+DHSFMB+DHSAMB)/AMASS

C  
C  
C

DERIVATIVES OF PITCH MOMENTS W/RESPECT TO THETA

DPBFTH=-AL5\*DHBPTH  
 DPBATH=AL6\*DHBATH  
 DPBTH=DPBFTH+DPBATH  
 DPPLTH=AL7\*(DHPFTH+DHPATH)  
 DPSFTH=-PBBAR\*(PLSF\*AL3\*WIDTH/SIN31-ASEAL1/2.0\*SIN31)  
 DPSATH=-PBBAR\*(PLSA\*AL4\*WIDTH/SIN32-ASEAL2/2.0\*SIN32)  
 DPSTH=(DPSFTH+DPSATH)  
 DTHTH=(DPBTH+DPPLTH+DPSTH)/AIYY

C

DPDMP=-AKDMP\*V  
 DTHDTH=DPDMP/AIYY

C  
C  
C  
C

DERIVATIVES OF PITCH MOMENTS W/RESPECT TO Z

DPBFZ=-AL5\*DHBFZ  
 DPBAZ=AL6\*DHBZ  
 DPPPZ=-XCP\*DHPBZ-ALDIFF\*HPRES/(2.0\*BUBUGT)  
 DPSFZ=PBBAR\*(PLSF\*WIDTH/SIN31-ASEAL1/2.0\*SIN31)  
 DPSAZ=-PBBAR\*(PLSA\*WIDTH/SIN31+ASEAL2/2.0\*SIN32)

SES01910  
 SES01920  
 SES01930  
 SES01940  
 SES01950  
 SES01960  
 SES01970  
 SES01980  
 SES01990  
 SES02000  
 SES02010  
 SES02020  
 SES02030  
 SES02040  
 SES02050  
 SES02060  
 SES02070  
 SES02080  
 SES02090  
 SES02100  
 SES02110  
 SES02120  
 SES02130  
 SES02140  
 SES02150  
 SES02160  
 SES02170  
 SES02180  
 SES02190  
 SES02200  
 SES02210  
 SES02220  
 SES02230  
 SES02240  
 SES02250  
 SES02260  
 SES02270  
 SES02280  
 SES02290  
 SES02300  
 SES02310  
 SES02320  
 SES02330  
 SES02340  
 SES02350  
 SES02360  
 SES02370  
 SES02380



```

DPSFPZ=ASEAL1*PLSF*DPBDZ
DPSAPZ=-ASEAL2*PLSA*DPBDZ
DPSFPZ=DPSFZ+DPSFPZ
DPSAZ=DPSAZ+DPSAPZ
DPSZ=(DPSFZ+DPSAZ)
DPBZ=DPBFZ+DPBAZ
DTHZ=(DPSZ+DPBZ+DPPBZ)/AIYY
C C C
DERIVATIVE OF PITCH MOMENT W/RESPECT TO MB
DPPBMB=-XCP*DHPBMB
DPSFMB=-PLSF*DHSFMB
DPSAMB=PLSA*DHSAMB
DTHMB=(DPPBMB+DPSFMB+DPSAMB)/AIYY
C C C
LIST THE VARIOUS SENSITIVITY COEFFICIENTS
IF (ILIST.NE.1) GO TO 12
WRITE(5,9640) DHBFZ,DHBZ,DHSFZ,DHSZ,DHSAZ,DHPBZ,DHBPTH,
1DHBATH,DHSFTH,DHSATH,DHPPTH,DHPATH,DHBBMB,
2DHSFMB,DHSAMB
WRITE(6,9645) DPBFTH,DPBATH,DPPLTH,DPSFTH,DPSATH,DPBZ,DPBAZ,
1DPSFZ,DPSAZ,DPPBZ,DPBMB,DPSFMB,DPSAMB,DHATH
WRITE(6,9650) DPBZ,DUMB,DMBB,DMBZ,DMBMB
WRITE(6,9630) DZZ,DZTH,DZMB,DHZZ,DTHTH,DTHMB,
1DMBZ,DMBMB,DPBDZ,DPBMB
C C C C C C C
PREPARE FOR TIME DOMAIN SOLUTION
COMPUTE VARIABLES FOR INTEGRATION SUBROUTINE
12 RANGE=TF-TI
ACCEL=ACCEL-WSTEP/AMASS
QPOINT(2)=ACCEL/G
H=RANGE/NSTEP
T=TI
NPASS=IFIX(FLOAT(NSTEP)/FLOAT(NPTS))
IP=1
C C C
LOAD INITIAL CONDITIONS INTO OUTPUT MATRIX
DO 15 I=1,NORD
OUTVEC(IP,I)=QPOINT(I)
15 CONTINUE
TIME(IP)=T
ICOUNT=NPTS
NPTS=1

```





C  
C  
C

COMMENCE THE TIME DOMAIN SOLUTION

```

25 MORD=5
   CALL DEQRKF (MORD)
   NPTS=NPTS+1
   QPOINT{1}=ALD0+X{1}*12.0
   QPOINT{2}=XDOT{2}/G
   QPOINT{3}={THEIAD+X{3})*RADEG
   QPOINT{4}=XDOT{4}*RADEG
   QPOINT{5}={AMB0+X{5}}/RHOA
   QPOINT{6}=PBBAR0+X{6}
   IP=IP+1
   TIME(IP)=T
   DO 30 I=1,NORD
   OUTVEC(IP,I)=QPOINT(I)
30  CONTINUE
   IF(NPTS.LT.(ICOUNT+1)) GO TO 25

```

C  
C  
C

OUTPUT THE TIME DOMAIN SOLUTION RESULTS

```

K=0
WRITE(6,9600)
WRITE(6,9605)
WRITE(6,9655)
DO 35 I=1,NPTS
K=K+1
WRITE(6,9660) TIME(I), (OUTVEC(I,J), J=1,NORD)
IF(K.LT.70) GO TO 35
WRITE(6,9600)
WRITE(6,9655)
K=0

```

35 CONTINUE

C  
C  
C

PREPARE TO PLOT OUTPUT DATA

```

DO 85 I=1,NORD
WRITE(6,9600)
DO 40 J=1,NPTS
PLOTV(J)=OUTVEC(J,I)
40 CONTINUE

```

C  
C

CHECK FOR PLOTP OR VERSATEC PLOTTING DESIRED

```

IF(IPL0T.NE.1) GO TO 44
IF(I.EQ.5) GO TO 85
CALL VERSAP (NPTS,TIME,PLOTV)
GO TO 35
44 CALL PLOTP (TIME,PLOTV,NPTS,0)

```

SES03280

SES02870  
SES02880  
SES02890  
SES02900  
SES02910  
SES02920  
SES02930  
SES02940  
SES02950  
SES02960  
SES02970  
SES02980  
SES02990  
SES03000  
SES03010  
SES03020  
SES03030  
SES03040  
SES03050  
SES03060  
SES03070  
SES03080  
SES03090  
SES03100  
SES03110  
SES03120  
SES03130  
SES03140  
SES03150  
SES03160  
SES03170  
SES03180  
SES03190  
SES03200  
SES03210  
SES03220  
SES03230  
SES03240  
SES03250  
SES03260  
SES03270  
SES03120  
SES03130



```

GO TO (45,50,55,60,65,70),I
45 WRITE(6,9665)
GO TO 80
50 WRITE(6,9670)
GO TO 80
55 WRITE(6,9675)
GO TO 80
60 WRITE(6,9680)
GO TO 80
65 WRITE(6,9685)
GO TO 80
70 WRITE(6,9695)
80 WRITE(6,9605)
85 CONTINUE
WRITE(6,9600)
9500 FORMAT(2F10.1,2I10)
9505 FORMAT(5F10.4)
9510 FORMAT(4F10.4)
9600 FORMAT(1H)
9605 FORMAT(//,20X,'TWO-DEGREE OF FREEDOM CAB SES LINEAR MODEL',/)
9610 FORMAT(//,1X,PI = ,F10.1//,1X,TF = ,F10.1//,1X,NO OF INTEGRATION STEPS = ,
2I10)
9615 FORMAT(//,1X,'OPERATING CONDITION',//,1X,
1'WEIGHT' = ,F10.4//,1X,'DRAFT' = ,F10.4//,1X,'PITCH' ,
2'ANGLE' = ,F10.4//,1X,'PLENUM PRESSURE' = ,F10.4//,1X,
3'SPEED' = ,F10.4)
9620 FORMAT(//,5X,'W',8X,'HBA',7X,'HSA',7X,
1'HPR',1X,'HPRES',5X,'RES',//,1X,9F10.4)
9625 FORMAT(//,8X,'PBA',7X,'PSF',7X,'PSA',6X,'PPLAN',6X,
1'PPRES',6X,'RES2',//,1X,7F10.4)
9630 FORMAT(//,1X,'DZZ' = ,E11.4//,1X,'DZTH' = ,E11.4//,1X,
2'E11.4//,1X,'DTHZ' = ,E11.4//,1X,'DTHMB' = ,E11.4,
3//,1X,'DMBZ' = ,E11.4//,1X,
4'DMBB' = ,E11.4//,1X,'DPMB' = ,E11.4)
5'DPBDZ' = ,E11.4//,1X,'HEAVE SENSITIVITY COEFFICIENTS',//,1X
9640 FORMAT(1X,'SENSITIVITY COEFFICIENTS',//,1X,
1'DHBFZ' = ,E11.4//,1X,'DHBZ' = ,E11.4//,1X,'DHSFZ' = ,
2'E11.4//,1X,'DHSZ' = ,E11.4//,1X,
3'DHPBZ' = ,E11.4//,1X,'DHPFTH' = ,E11.4//,1X,'DHSATH' = ,
4//,1X,'DHBATH' = ,E11.4//,1X,'DHPATH' = ,E11.4//,1X,
7'DHPMB' = ,E11.4//,1X,'DHSFMB' = ,E11.4//,1X,'DHSAMB' = ,
8E11.4)
9645 FORMAT(//,1X,'PITCH SENSITIVITY COEFFICIENTS',//,1X,
1'DPBFTH' = ,E11.4//,1X,'DPBATH' = ,E11.4//,1X,
2'DPPLTH' = ,E11.4//,1X,'DPSPFH' = ,E11.4//,1X,

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XDOT(4)=DTHZ*X(1)+DPTH*X(3)+DTMB*X(5)+DTHDTH*X(4)+PACCEL
XDOT(5)=DMBZ*X(1)+DMBMB*X(5)
X(6)=DPBDZ*X(1)+DPBMB*X(5)
RETURN
END
SUBROUTINE VERSAP (NUMPTS,X,Y)
DIMENSION X(300),Y(300),RTB(12),ITB(28)
EQUIVALENCE (FILE,RTB(5))
READ*8 TITLE(12)
READ(5,9515) TITLE
FORMAT(6A8)
RTB(1)=0.0
RTB(2)=0.0
RTB(3)=0.0
RTB(4)=0.0
DO 1 I=1,12
1 ITB(I)=0
ITB(3)=6
ITB(4)=7
CALL DRAWP (NUMPTS,X,Y,ITB,RTB)
RETURN
END

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SES04730  
 SES04740  
 SES04750  
 SES04760  
 SES04770



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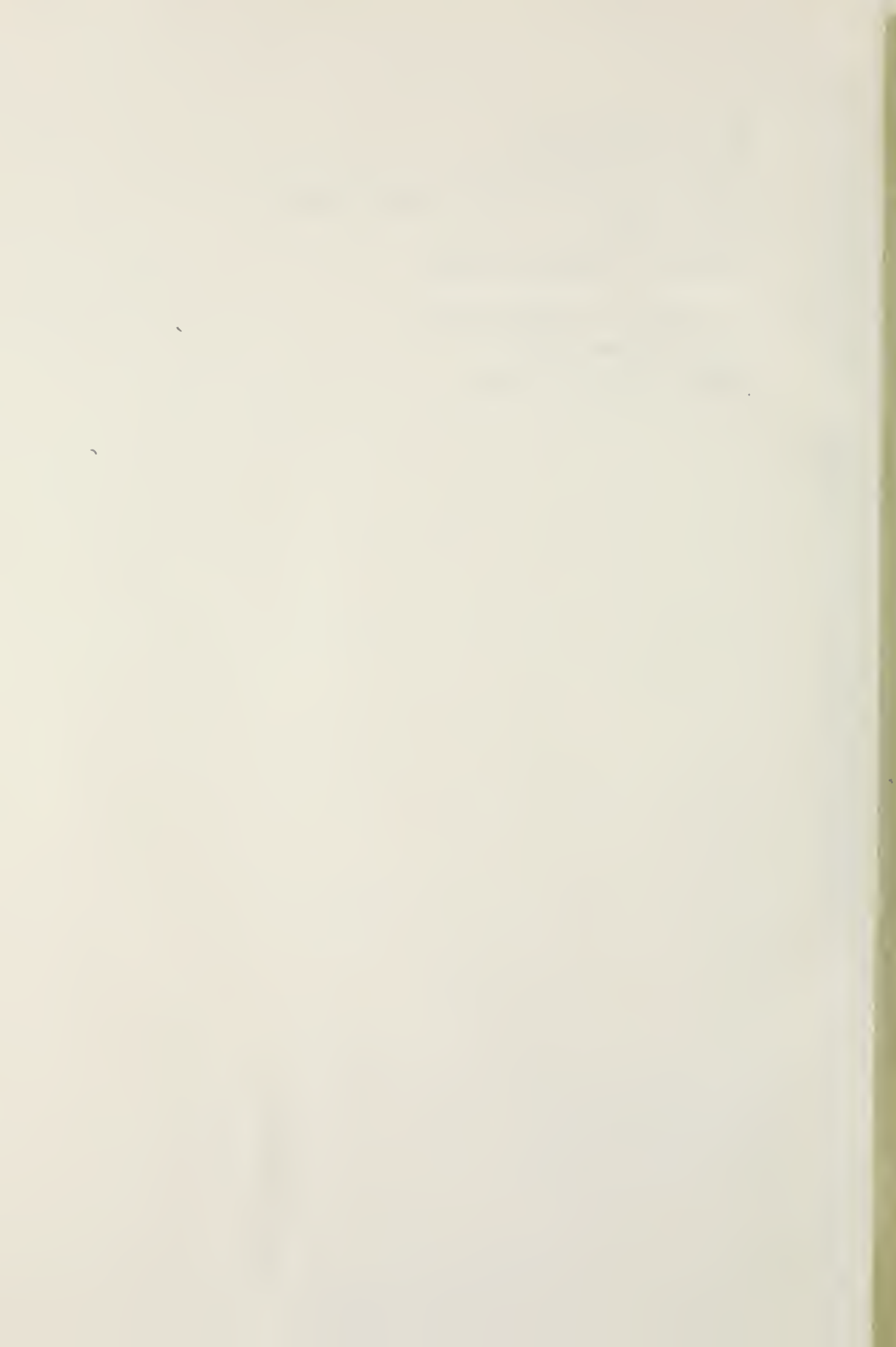
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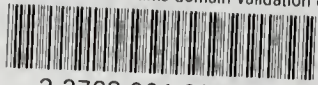
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